

# BJT Device Equations and Small-Signal Models

## Notation

The notations used here for voltages and currents correspond to the following conventions: Dc bias values are indicated by an upper case letter with upper case subscripts, e.g.  $V_{DS}$ ,  $I_C$ . Instantaneous values of small-signal variables are indicated by a lower-case letter with lower-case subscripts, e.g.  $v_s$ ,  $i_c$ . Total values are indicated by a lower-case letter with upper-case subscripts, e.g.  $v_{BE}$ ,  $i_D$ . Circuit symbols for independent sources are circular and symbols for controlled sources have a diamond shape. Voltage sources have a  $\pm$  sign within the symbol and current sources have an arrow.

## Device Equations

Figure 1 shows the circuit symbols for the npn and pnp BJTs. In the active mode, the collector-base junction is reverse biased and the base-emitter junction is forward biased. For the npn device, the active-mode collector and base currents are given by

$$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right) \quad i_B = \frac{i_C}{\beta} \quad (1)$$

where  $V_T$  is the thermal voltage,  $I_S$  is the saturation current, and  $\beta$  is the base-to-collector current gain. These are given by

$$V_T = \frac{kT}{q} = 0.025 \text{ V for } T = 290 \text{ K} = 25.86 \text{ mV for } T = 300 \text{ K} \quad (2)$$

$$I_S = I_{S0} \left(1 + \frac{v_{CE}}{V_A}\right) \quad (3)$$

$$\beta = \beta_0 \left(1 + \frac{v_{CE}}{V_A}\right) \quad (4)$$

where  $V_A$  is the Early voltage and  $I_{S0}$  and  $\beta_0$ , respectively, are the zero bias values of  $I_S$  and  $\beta$ . Because  $I_S/\beta = I_{S0}/\beta_0$ , it follows that  $i_B$  is not a function of  $v_{CE}$ . The equations apply to the pnp device if the subscripts  $BE$  and  $CE$  are reversed.

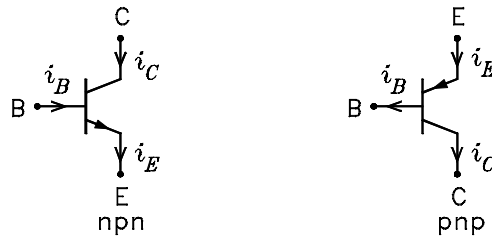


Figure 1: BJT circuit symbols.

The emitter-to-collector current gain  $\alpha$  is defined as the ratio  $i_C/i_E$ . To solve for this, we can write

$$i_E = i_B + i_C = \left(\frac{1}{\beta} + 1\right) i_C = \frac{1 + \beta}{\beta} i_C \quad (5)$$

It follows that

$$\alpha = \frac{i_C}{i_E} = \frac{\beta}{1 + \beta} \quad (6)$$

Thus the currents are related by the equations

$$i_C = \beta i_B = \alpha i_E \quad (7)$$

## Transfer and Output Characteristics

The transfer characteristics are a plot of the collector current  $i_C$  as a function of the base-to-emitter voltage  $v_{BE}$  with the collector-to-emitter voltage  $v_{CE}$  held constant. From Eqs. 1 and 3, we can write

$$i_C = I_{S0} \left( 1 + \frac{v_{CE}}{V_A} \right) \exp \left( \frac{v_{BE}}{V_T} \right) \quad (8)$$

It follows that  $i_C$  varies exponentially with  $v_{BE}$ . A plot of this variation is given in Fig. 2. It can be seen from the plot that the collector current is essentially zero until the base-to-emitter voltage reaches a threshold value. Above this value, the collector current increases rapidly. The threshold value is typically in the range of 0.5 to 0.6 V. For high current transistors, it is usually smaller. The plot shows a single curve. If  $v_{CE}$  is increased, the current for a given  $v_{BE}$  is larger. However, the displacement between the curves is so small that it can be difficult to distinguish between them. The small-signal transconductance  $g_m$  defined below is the slope of the transfer characteristics curve evaluated at the quiescent or dc operating point for a device.

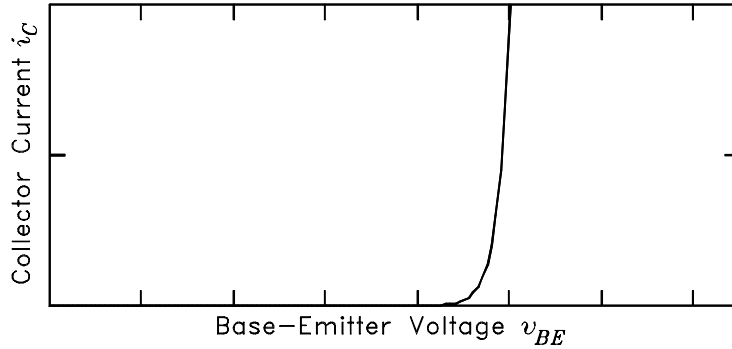


Figure 2: BJT transfer characteristics.

The output characteristics are a plot of the collector current  $i_C$  as a function of the collector-to-emitter voltage  $v_{CE}$  with the base current  $i_B$  held constant. From Eqs. 1 and 4, we can write

$$i_C = \beta_0 \left( 1 + \frac{v_{CE}}{V_A} \right) i_B \quad (9)$$

It follows that  $i_C$  varies linearly with  $v_{CE}$ . A plot of this variation is given in Fig. 3. For small  $v_{CE}$  such that  $0 \leq v_{CE} < v_{BE}$ , Eq. (9) does not hold. In the region in Fig. 3 where this holds, the bjt is saturated. The small-signal collector-to-emitter resistance  $r_0$  defined below is the reciprocal of the slope of the transfer characteristics curve evaluated at the quiescent or dc operating point for a device.

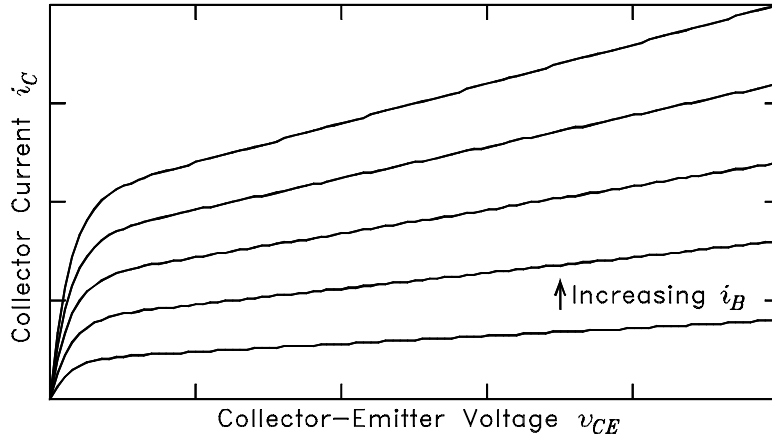


Figure 3: BJT output characteristics.

## Bias Equation

Figure 4(a) shows the BJT with the external circuits represented by Thévenin dc circuits. If the BJT is biased in the active region, we can write

$$\begin{aligned} V_{BB} - V_{EE} &= I_B R_{BB} + V_{BE} + I_E R_{EE} \\ &= \frac{I_C}{\beta} R_{BB} + V_{BE} + \frac{I_C}{\alpha} R_{EE} \end{aligned} \quad (10)$$

This equation can be solved for  $I_C$  to obtain

$$I_C = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/\beta + R_{EE}/\alpha} \quad (11)$$

It can be seen from Fig. 2 that large changes in  $I_C$  are associated with small changes in  $V_{BE}$ . This makes it possible to calculate  $I_C$  by assuming typical values of  $V_{BE}$ . Values in the range from 0.6 V to 0.7 V are commonly used. Two sets of values for  $\alpha$  and  $\beta$  are convenient for hand calculations. One is  $\alpha = 0.99$  and  $\beta = 99$ . The other is  $\alpha = 0.995$  and  $\beta = 199$ .

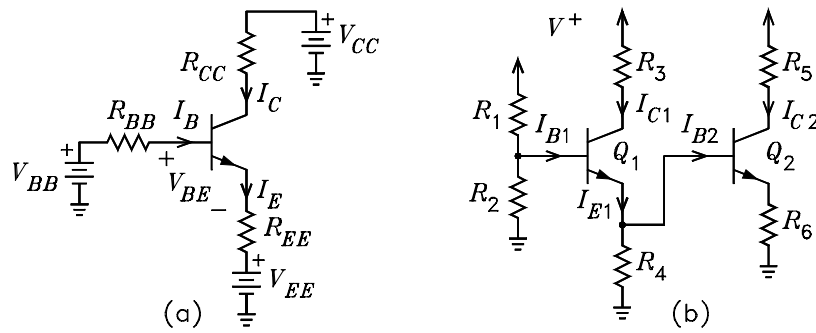


Figure 4: (a) BJT dc bias circuit. (b) Circuit for Example 1.

**Example 1** Figure 4(b) shows a BJT dc bias circuit. It is given that  $V^+ = 15$  V,  $R_1 = 20$  k $\Omega$ ,  $R_2 = 10$  k $\Omega$ ,  $R_3 = R_4 = 3$  k $\Omega$ ,  $R_5 = R_6 = 2$  k $\Omega$ . Solve for  $I_{C1}$  and  $I_{C2}$ . Assume  $V_{BE} = 0.7$  V and  $\beta = 100$  for each transistor.

*Solution.* For  $Q_1$ , we have  $V_{BB1} = V^+ R_2 / (R_1 + R_2)$ ,  $R_{BB1} = R_1 \parallel R_2$ ,  $V_{EE1} = -I_{B2} R_4 = -I_{C2} R_4 / \beta$ ,  $V_{EE1} = 0$ , and  $R_{EE1} = R_4$ . For  $Q_2$ , we have  $V_{BB2} = I_{E1} R_4 = I_{C1} R_4 / \alpha$ ,  $R_{BB2} = R_4$ ,  $V_{EE2} = 0$ ,  $R_{EE2} = R_6$ . Thus the bias equations are

$$V^+ \frac{R_2}{R_1 + R_2} + \frac{I_{C2}}{\beta} R_4 = V_{BE} + \frac{I_{C1}}{\beta} R_1 \parallel R_2 + \frac{I_{C1}}{\alpha} R_4$$

$$\frac{I_{C1}}{\alpha} R_4 = V_{BE} + \frac{I_{C2}}{\beta} R_4 + \frac{I_{C2}}{\alpha} R_6$$

These equations can be solved simultaneously to obtain  $I_{C1} = 1.41 \text{ mA}$  and  $I_{C2} = 1.74 \text{ mA}$ .

## Hybrid- $\pi$ Model

Let each current and voltage be written as the sum of a dc component and a small-signal ac component as follows:

$$i_C = I_C + i_c \quad i_B = I_B + i_b \quad (12)$$

$$v_{BE} = V_{BE} + v_{be} \quad v_{CE} = V_{CE} + v_{ce} \quad (13)$$

If the ac components are sufficiently small, we can write

$$i_c = \frac{\partial I_C}{\partial V_{BE}} v_{be} + \frac{\partial I_C}{\partial V_{CE}} v_{ce} \quad i_b = \frac{\partial I_B}{\partial V_{BE}} v_{be} \quad (14)$$

where the derivatives are evaluated at the dc bias values. Let us define the transconductance  $g_m$ , the collector-to-emitter resistance  $r_0$ , and the base-to-emitter resistance  $r_\pi$  as follows:

$$g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{I_S}{V_T} \exp\left(\frac{V_{BE}}{V_T}\right) = \frac{I_C}{V_T} \quad (15)$$

$$r_0 = \left(\frac{\partial I_C}{\partial V_{CE}}\right)^{-1} = \left[\frac{I_{S0}}{V_A} \exp\left(\frac{V_{BE}}{V_T}\right)\right]^{-1} = \frac{V_A + V_{CE}}{I_C} \quad (16)$$

$$r_\pi = \left(\frac{\partial I_B}{\partial V_{BE}}\right)^{-1} = \left[\frac{I_{S0}}{\beta_0 V_T} \exp\left(\frac{V_{BE}}{V_T}\right)\right]^{-1} = \frac{V_T}{I_B} \quad (17)$$

The collector and base currents can thus be written

$$i_c = i'_c + \frac{v_{ce}}{r_0} \quad i_b = \frac{v_\pi}{r_\pi} \quad (18)$$

where

$$i'_c = g_m v_\pi \quad v_\pi = v_{be} \quad (19)$$

The small-signal circuit which models these equations is given in Fig. 5(a). This is called the hybrid- $\pi$  model. The resistor  $r_x$ , which does not appear in the above equations, is called the base spreading resistance. It represents the resistance of the connection to the base region inside the device. Because the base region is very narrow, the connection exhibits a resistance which often cannot be neglected.

The small-signal base-to-collector ac current gain  $\beta$  is defined as the ratio  $i'_c/i_b$ . It is given by

$$\beta = \frac{i'_c}{i_b} = \frac{g_m v_\pi}{i_b} = g_m r_\pi = \frac{I_C}{V_T} \frac{V_T}{I_B} = \frac{I_C}{I_B} \quad (20)$$

Note that  $i_c$  differs from  $i'_c$  by the current through  $r_0$ . Therefore,  $i_c/i_b \neq \beta$  unless  $r_0 = \infty$ .

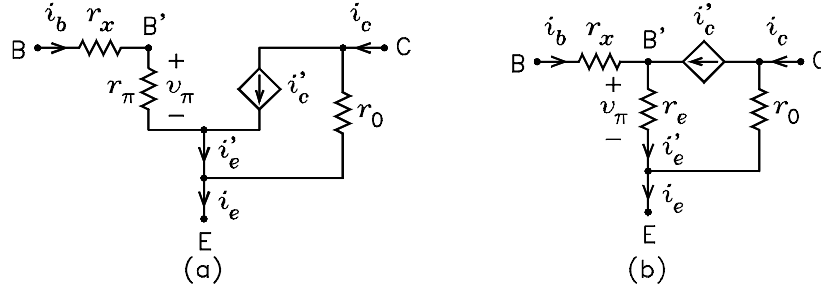


Figure 5: (a) Hybrid- $\pi$  model. (b) T model.

## T Model

The T model replaces the resistor  $r_\pi$  in series with the base with a resistor  $r_e$  in series with the emitter. This resistor is called the emitter intrinsic resistance. The current  $i'_e$  can be written

$$i'_e = i_b + i'_c = \left(\frac{1}{\beta} + 1\right) i'_c = \frac{1 + \beta}{\beta} i'_c = \frac{i'_c}{\alpha} \quad (21)$$

where  $\alpha$  is the small-signal emitter-to-collector ac current gain given by

$$\alpha = \frac{\beta}{1 + \beta} \quad (22)$$

Thus the current  $i'_c$  can be written

$$i'_c = \alpha i'_e \quad (23)$$

The voltage  $v_\pi$  can be related to  $i'_e$  as follows:

$$v_\pi = i_b r_\pi = \frac{i'_c}{\beta} r_\pi = \frac{\alpha i'_e}{\beta} r_\pi = i'_e \frac{\alpha r_\pi}{\beta} = i'_e \frac{r_\pi}{1 + \beta} = i'_e r_e \quad (24)$$

It follows that the intrinsic emitter resistance  $r_e$  is given by

$$r_e = \frac{v_\pi}{i'_e} = \frac{r_\pi}{1 + \beta} = \frac{V_T}{(1 + \beta) I_B} = \frac{V_T}{I_E} \quad (25)$$

The T model of the BJT is shown in Fig. 5(b). The currents in both models are related by the equations

$$i'_c = g_m v_\pi = \beta i_b = \alpha i'_e \quad (26)$$

## Simplified T Model

Figure 6 shows the T model with a Thévenin source in series with the base. We wish to solve for an equivalent circuit in which the source  $i'_c$  connects from the collector node to ground rather than from the collector node to the B' node.

The first step is to replace the source  $\alpha i'_e$  with two identical series sources with the common node grounded. The circuit is shown in Fig. 7(a). The object is to absorb the left  $\alpha i'_e$  source into the base-emitter circuit. For the circuit, we can write

$$v_e = v_{tb} - \frac{i'_e}{1 + \beta} (R_{tb} + r_x) - i'_e r_e = v_{tb} - i'_e \left( \frac{R_{tb} + r_x}{1 + \beta} + r_e \right) \quad (27)$$

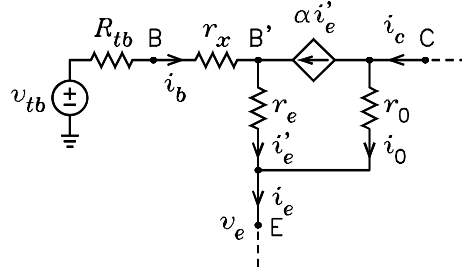


Figure 6: T model with Thévenin source connected to the base.

Let us define the resistance  $r'_e$  by

$$r'_e = \frac{R_{tb} + r_x}{1 + \beta} + r_e \stackrel{\text{or}}{=} \frac{R_{tb} + r_x + r_\pi}{1 + \beta} \quad (28)$$

With this definition,  $v_e$  is given by

$$v_e = v_{tb} - i'_e r'_e \quad (29)$$

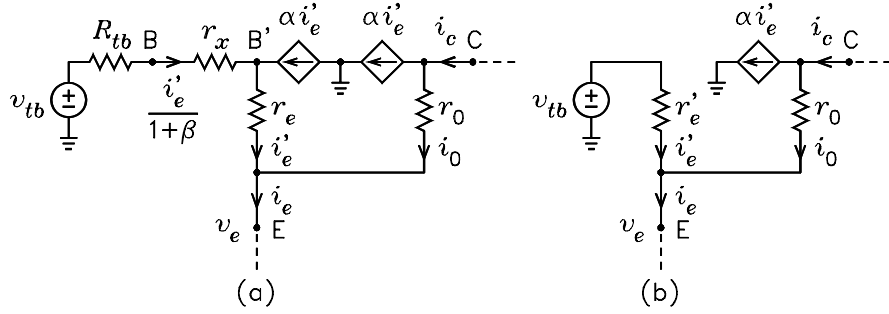


Figure 7: (a) Circuit with the  $i'_e$  source replaced by identical series sources. (b) Simplified T model.

The circuit which models Eq. (29) is shown in Fig. 7(b). We will call this the simplified T model. It predicts the same emitter and collector currents as the circuit in Fig. 6. Note that the resistors  $R_{tb}$  and  $r_x$  do not appear in this circuit because they are contained in  $r'_e$ .

## Norton Collector Circuit

The Norton equivalent circuit seen looking into the collector can be used to solve for the response of the common-emitter and common-base stages. It consists of a parallel current source  $i_{c(sc)}$  and a resistor  $r_{ic}$  from the collector to signal ground. Fig. 8(a) shows the BJT with Thévenin sources connected to its base and emitter. To solve for the Norton equivalent circuit seen looking into the collector, we use the simplified T model in Fig. 8(b).

By superposition of  $v_c$ ,  $\alpha i'_e$ ,  $v_{tb}$ , and  $v_{te}$ , the following equations for  $i_c$  and  $i'_e$  can be written

$$i_c = \frac{v_c}{r_0 + r'_e \parallel R_{te}} + \alpha i'_e - \frac{v_{tb}}{r'_e + R_{te} \parallel r_0} \frac{R_{te}}{R_{te} + r_0} - \frac{v_{te}}{R_{te} + r'_e \parallel r_0} \frac{r'_e}{r'_e + r_0} \quad (30)$$

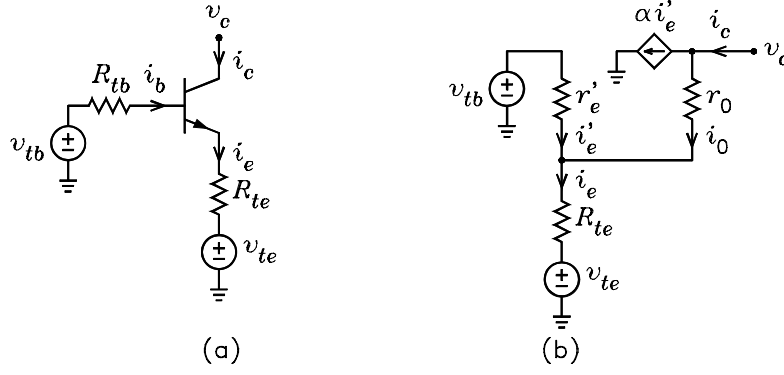


Figure 8: (a) BJT with Thevenin sources connected to the base and the emitter. (b) Simplified T model.

$$i'_e = \frac{v_{tb}}{r'_e + R_{te} \parallel r_0} - \frac{v_{te}}{R_{te} + r'_e \parallel r_0} - \frac{v_c}{r_0 + r'_e \parallel R_{te}} \frac{R_{te}}{r'_e + R_{te}} \quad (31)$$

These can be solved to obtain

$$i_c = \frac{v_{tb}}{r'_e + R_{te} \parallel r_0} \left( \alpha - \frac{R_{te}}{R_{te} + r_0} \right) - \frac{v_{te}}{R_{te} + r'_e \parallel r_0} \left( \alpha + \frac{r'_e}{r'_e + r_0} \right) + \frac{v_c}{r_0 + r'_e \parallel R_{te}} \left( 1 - \frac{\alpha R_{te}}{r'_e + R_{te}} \right) \quad (32)$$

This equation is of the form

$$i_c = i_{c(sc)} + \frac{v_c}{r_{ic}} \quad (33)$$

where  $i_{c(sc)}$  and  $r_{ic}$  are given by

$$i_{c(sc)} = G_{mb} v_{tb} - G_{me} v_{te} \quad (34)$$

$$r_{ic} = \frac{r_0 + r'_e \parallel R_{te}}{1 - \alpha R_{te} / (r'_e + R_{te})} \quad (35)$$

and  $G_{mb}$  and  $G_{me}$  are given by

$$G_{mb} = \frac{1}{r'_e + R_{te} \parallel r_0} \left( \alpha - \frac{R_{te}}{R_{te} + r_0} \right) \quad (36)$$

$$\stackrel{\text{or}}{=} \frac{\alpha}{r'_e + R_{te} \parallel r_0} \frac{r_0 - R_{te} / \beta}{r_0 + R_{te}}$$

$$G_{me} = \frac{1}{R_{te} + r'_e \parallel r_0} \left( \alpha + \frac{r'_e}{r'_e + r_0} \right) \quad (37)$$

$$\stackrel{\text{or}}{=} \frac{\alpha}{r'_e + R_{te} \parallel r_0} \frac{r_0 + r'_e / \alpha}{r_0 + R_{te}}$$

The Norton equivalent circuit seen looking into the collector is shown in Fig. 9.

For the case  $r_0 \gg R_{te}$  and  $r_0 \gg r'_e$ , we can write

$$i_{c(sc)} = G_m (v_{tb} - v_{te}) \quad (38)$$

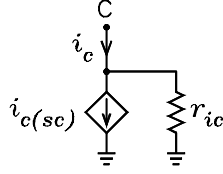


Figure 9: (a) Circuit for calculating  $r_{ic}$ . (b) Norton collector circuit.

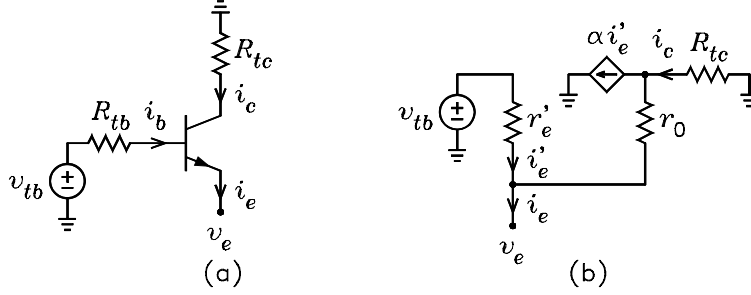


Figure 10: (a) BJT with Thévenin source connected to the base. (b) T model circuit for calculating  $v_{e(oc)}$ .

where

$$G_m = \frac{\alpha}{r'_e + R_{te}} \quad (39)$$

The value of  $i_{c(sc)}$  calculated with this approximation is simply the value of  $\alpha i'_e$ , where  $i'_e$  is calculated with  $r_0$  considered to be an open circuit. The term “ $r_0$  approximations” is used in the following when  $r_0$  is neglected in calculating  $i_{c(sc)}$  but not neglected in calculating  $r_{ic}$ .

## Thévenin Emitter Circuit

The Thévenin equivalent circuit seen looking into the emitter is useful in calculating the response of common-collector stages. It consists of a voltage source  $v_{e(oc)}$  in series with a resistor  $r_{ie}$  from the emitter node to signal ground. Fig. 10(a) shows the BJT symbol with a Thévenin source connected to the base. The resistor  $R_{tc}$  represents the external load resistance in series with the collector. To solve for the Thévenin equivalent circuit seen looking into the emitter, we use the simplified T model in Fig. 10(b).

By superposition of  $v_{tb}$ ,  $i_e$ , and  $\alpha i'_e$ , the following equations for  $v_e$  and  $i'_e$  can be written

$$\begin{aligned} v_e = & v_{tb} \frac{r_0 + R_{tc}}{r'_e + r_0 + R_{tc}} - i_e [r'_e \parallel (r_0 + R_{tc})] \\ & - \alpha i'_e \frac{R_{tc} r'_e}{r'_e + r_0 + R_{tc}} \end{aligned} \quad (40)$$

$$i'_e = \frac{v_{tb} - v_e}{r'_e} \quad (41)$$



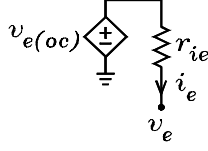


Figure 11: (a) Circuit for calculating  $i_{e(sc)}$ . (b) Thévenin emitter circuit.

These can be solved to obtain

$$v_e \left( 1 - \frac{\alpha R_{tc}}{r'_e + r_0 + R_{tc}} \right) = \frac{v_{tb}}{r'_e + r_0 + R_{tc}} [r_0 + (1 - \alpha) R_{tc}] - i_e r'_e \parallel (r_0 + R_{tc}) \quad (42)$$

which simplifies to

$$v_e = v_{tb} \frac{r_0 + (1 - \alpha) R_{tc}}{r'_e + r_0 + (1 - \alpha) R_{tc}} - i_e \frac{r'_e (r_0 + R_{tc})}{r'_e + r_0 + (1 - \alpha) R_{tc}} \quad (43)$$

This equation is of the form

$$v_e = v_{e(oc)} - i_e r_{ie} \quad (44)$$

where  $v_{e(oc)}$  and  $r_{ie}$  are given by

$$\begin{aligned} v_{e(oc)} &= v_{tb} \frac{r_0 + (1 - \alpha) R_{tc}}{r'_e + r_0 + (1 - \alpha) R_{tc}} \\ &\equiv v_{tb} \frac{r_0 + R_{tc} / (1 + \beta)}{r'_e + r_0 + R_{tc} / (1 + \beta)} \end{aligned} \quad (45)$$

$$\begin{aligned} r_{ie} &= r'_e \frac{r_0 + R_{tc}}{r'_e + r_0 + (1 - \alpha) R_{tc}} \\ &\equiv r'_e \frac{r_0 + R_{tc}}{r'_e + r_0 + R_{tc} / (1 + \beta)} \end{aligned} \quad (46)$$

The Thévenin equivalent circuit seen looking into the emitter is shown in Fig. ??.

## Thévenin Base Circuit

Although the base is not an output terminal, the Thévenin equivalent circuit seen looking into the base is useful in calculating the base current. It consists of a voltage source  $v_{b(oc)}$  in series with a resistor  $r_{ib}$  from the base node to signal ground. Fig. 12(a) shows the BJT symbol with a Thévenin source connected to its emitter. Fig. 12(b) shows the Pi model for calculating the base voltage.

By superposition of  $v_{te}$ ,  $i_b$ , and  $\beta i_b$ , treating each branch of  $\beta i_b$  separately in the superposition, we can write the following equation for  $v_b$

$$\begin{aligned} v_b &= v_{te} \frac{r_0 + R_{tc}}{R_{te} + r_0 + R_{tc}} + i_b [r_x + r_\pi + R_{te} \parallel (r_0 + R_{tc})] \\ &\quad + \beta i_b \frac{r_0 R_{te}}{R_{te} + r_0 + R_{tc}} \\ &= v_{te} \frac{r_0 + R_{tc}}{R_{te} + r_0 + R_{tc}} + i_b \left\{ r_x + r_\pi + R_{te} \parallel (r_0 + R_{tc}) \right. \\ &\quad \left. + \beta \frac{r_0 R_{te}}{R_{te} + r_0 + R_{tc}} \right\} \end{aligned} \quad (47)$$

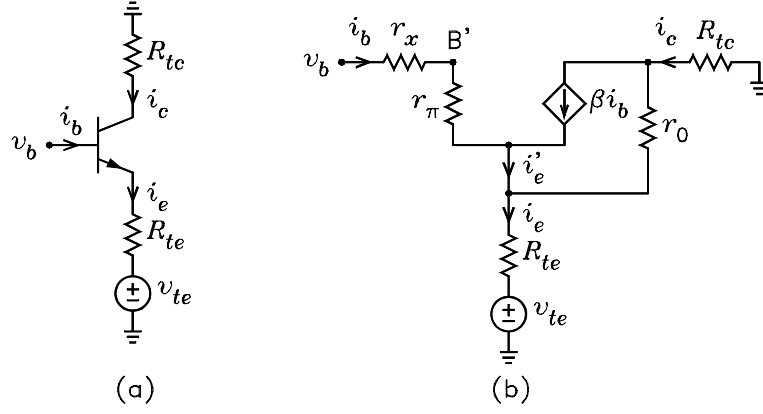


Figure 12: (a) BJT with Thevenin source connected to the emitter. (b) T model for calculating  $v_{b(oc)}$ .

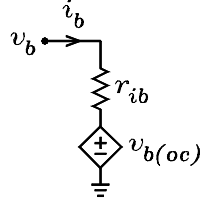


Figure 13: (a) Circuit for calculating  $v_b$ . (b) Thévenin base circuit.

This equation is of the form

$$v_b = v_{b(oc)} + i_b r_{ib} \quad (48)$$

where  $v_{b(oc)}$  and  $r_{ib}$  are given by

$$v_{b(oc)} = v_{te} \frac{r_0 + R_{tc}}{R_{te} + r_0 + R_{tc}} \quad (49)$$

$$\begin{aligned} r_{ib} &= r_x + r_\pi + R_{te} \parallel (r_0 + R_{tc}) \\ &\quad + \beta \frac{r_0 R_{te}}{R_{te} + r_0 + R_{tc}} \\ &\stackrel{\text{or}}{=} r_x + (1 + \beta) r_e + R_{te} \parallel (r_0 + R_{tc}) \\ &\quad + \beta \frac{r_0 R_{te}}{R_{te} + r_0 + R_{tc}} \end{aligned} \quad (50)$$

The equivalent circuit which models these equations is shown in Fig. 13.

## The $r_0$ Approximations

The  $r_0$  approximations approximate  $r_0$  as an open circuit in all equations except the one for  $r_{ic}$ . In this case, the simplified T model reduces to the one shown in Fig. 14. Because  $r_0$  no longer connects to the emitter, there is only one emitter current and  $i'_e = i_e$ . This circuit is very useful in

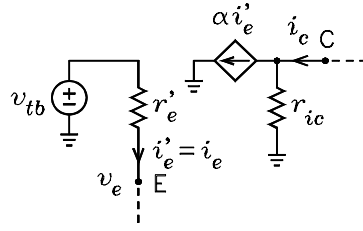


Figure 14: Emitter equivalent circuit with the  $r_0$  approximations.

making rapid hand calculations and gives little error in most applications. If  $r_0 = \infty$ , then  $r_{ic}$  is an open circuit.

The equations for the Norton collector circuit and the Thévenin emitter and base circuits are summarized as follows:

#### Norton Collector Circuit

$$i_{c(sc)} = \alpha i_e = G_m (v_{tb} - v_{te}) \quad G_m = \frac{\alpha}{r'_e + R_{te}} \quad (51)$$

$$r_{ic} = \frac{r_0 + r'_e \parallel R_{te}}{1 - \alpha R_{te} / (r'_e + R_{te})} \quad (52)$$

#### Thévenin Emitter Circuit

$$v_{e(oc)} = v_{tb} \quad r_{ie} = r'_e \quad (53)$$

#### Thévenin Base Circuit

$$v_{b(oc)} = v_{te} \quad r_{ib} = r_x + r_\pi + (1 + \beta) R_{te} \stackrel{\text{or}}{=} r_x + (1 + \beta) (r_e + R_{te}) \quad (54)$$

## Summary of Models

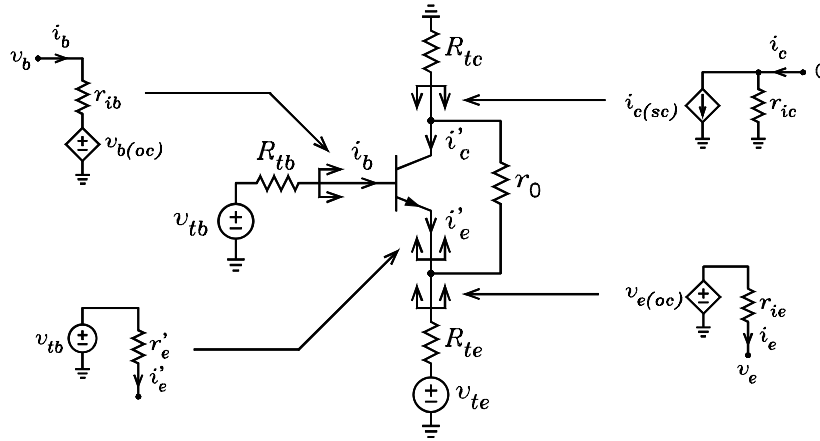


Figure 15: Summary of the small-signal equivalent circuits.

## Example Amplifier Circuits

This section describes several examples which illustrate the use of the small-signal equivalent circuits derived above to write by inspection the voltage gain, the input resistance, and the output resistance of both single-stage and two-stage amplifiers.

### The Common-Emitter Amplifier

Figure 16(a) shows the ac signal circuit of a common-emitter amplifier. We assume that the bias solution and the small-signal resistances  $r'_e$  and  $r_0$  are known. The output voltage and output resistance can be calculated by replacing the circuit seen looking into the collector by the Norton equivalent circuit of Fig. 9(b). With the aid of this circuit, we can write

$$v_o = -i_{c(sc)} (r_{ic} \parallel R_{tc}) = -G_{mb} (r_{ic} \parallel R_{tc}) v_{tb} \quad (55)$$

$$r_{out} = r_{ic} \parallel R_{tc} \quad (56)$$

where  $G_{mb}$  and  $r_{ic}$ , respectively, are given by Eqs. (??) and (??). The input resistance is given by

$$r_{in} = R_{tb} + r_{ib} \quad (57)$$

where  $r_{ib}$  is given by Eq. (??).

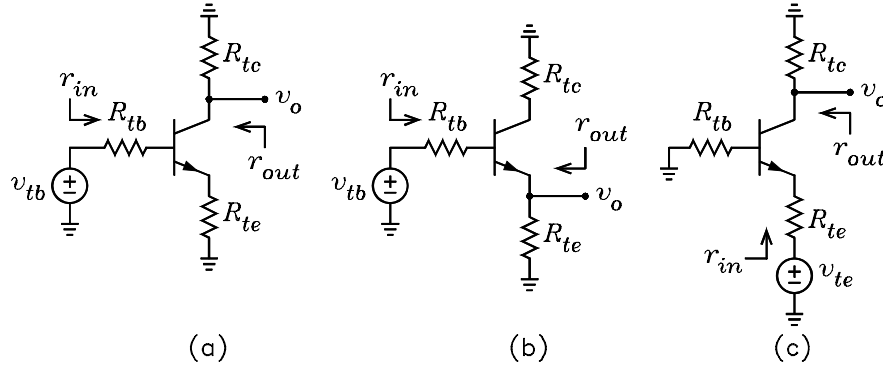


Figure 16: (a) Common-emitter amplifier. (b) Common-collector amplifier. (c) Common-base amplifier.

### The Common-Collector Amplifier

Figure 16(b) shows the ac signal circuit of a common-collector amplifier. We assume that the bias solution and the small-signal resistances  $r'_e$  and  $r_0$  are known. The output voltage and output resistance can be calculated by replacing the circuit seen looking into the emitter by the Thévenin equivalent circuit of Fig. 10(b). With the aid of this circuit, we can write

$$v_o = v_{e(oc)} \frac{R_{te}}{r_{ie} + R_{te}} = \frac{r_0 + R_{tc} / (1 + \beta)}{r'_e + r_0 + R_{tc} / (1 + \beta)} \frac{R_{te}}{r_{ie} + R_{te}} v_{tb} \quad (58)$$

$$r_{out} = r_{ie} \parallel R_{te} \quad (59)$$

where  $r_{ie}$  is given by Eq. (??). The input resistance is given by

$$r_{in} = R_{tb} + r_{ib} \quad (60)$$

where  $r_{ib}$  is given by Eq. (??).

## The Common-Base Amplifier

Figure 16(c) shows the ac signal circuit of a common-base amplifier. We assume that the bias solution and the small-signal parameters  $r'_e$  and  $r_0$  are known. The output voltage and output resistance can be calculated by replacing the circuit seen looking into the collector by the Norton equivalent circuit of Fig. 9(b). The input resistance can be calculated by replacing the circuit seen looking into the emitter by the Thévenin equivalent circuit of Fig. 10(b) with  $v_{e(oc)} = 0$ . With the aid of these circuits, we can write

$$v_o = -i_{c(sc)} (r_{ic} \parallel R_{tc}) = G_{me} (r_{ic} \parallel R_{tc}) v_{te} \quad (61)$$

$$r_{out} = r_{ic} \parallel R_{tc} \quad (62)$$

$$r_{in} = R_{te} + r_{ie} \quad (63)$$

where  $G_{me}$ ,  $r_{ic}$ , and  $r_{ie}$ , respectively, are given by Eqs. (??), (??), and (??).

## The CE/CC Amplifier

Figure 17(a) shows the ac signal circuit of a two-stage amplifier consisting of a CE stage followed by a CC stage. Such a circuit is used to obtain a high voltage gain and a low output resistance. The voltage gain can be written

$$\begin{aligned} \frac{v_o}{v_{tb1}} &= \frac{i_{c1(sc)}}{v_{tb1}} \times \frac{v_{tb2}}{i_{c1(sc)}} \times \frac{v_{e2(oc)}}{v_{tb2}} \times \frac{v_o}{v_{e2(oc)}} \\ &= G_{mb1} \times [-(r_{ic1} \parallel R_{C1})] \times \frac{r_0}{r'_{e2} + r_0} \times \frac{R_{te2}}{r_{ie2} + R_{te2}} \end{aligned} \quad (64)$$

where  $r'_{e2}$  is calculated with  $R_{tb2} = r_{ic1} \parallel R_{C1}$ . The input and output resistances are given by

$$r_{in} = R_{tb1} + r_{ib1} \quad (65)$$

$$r_{out} = r_{ie2} \parallel R_{te2} \quad (66)$$

Although not a part of the solution, the resistance seen looking out of the collector of  $Q_1$  is  $R_{tc1} = R_{C1} \parallel r_{ib2}$ .

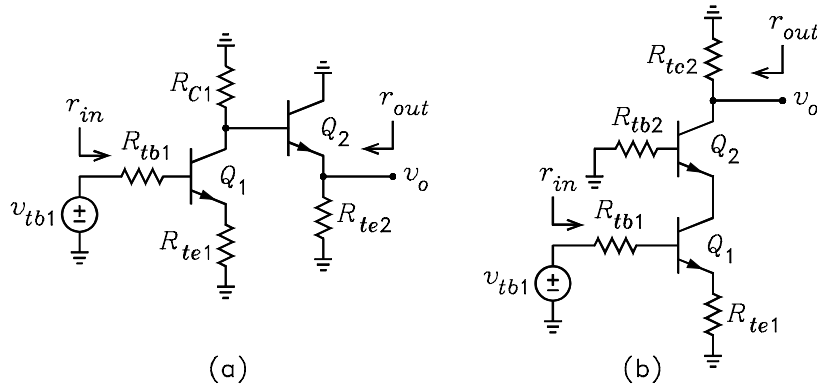


Figure 17: (a) CE-CC amplifier. (b) Cascode amplifier.

## The Cascode Amplifier

Figure 17(b) shows the ac signal circuit of a cascode amplifier. The voltage gain can be written

$$\begin{aligned}\frac{v_o}{v_{tb1}} &= \frac{i_{c1(sc)}}{v_{tb1}} \times \frac{v_{te2}}{i_{c1(sc)}} \times \frac{i_{c2(sc)}}{v_{te2}} \times \frac{v_o}{i_{c2(sc)}} \\ &= G_{m1} \times (-r_{ic1}) \times (-G_{me2}) \times (-r_{ic2} \parallel R_{tc2})\end{aligned}$$

where  $G_{me2}$  and  $r_{ic2}$  are calculated with  $R_{te2} = r_{ic1}$ . The input and output resistances are given by

$$r_{in} = R_{tb1} + r_{ib1}$$

$$r_{out} = R_{tc2} \parallel r_{ic2}$$

The resistance seen looking out of the collector of  $Q_1$  is  $R_{tc1} = r_{ie2}$ .

A second cascode amplifier is shown in Fig. 18(a) where a pnp transistor is used for the second stage. The voltage gain is given by

$$\begin{aligned}\frac{v_o}{v_{tb1}} &= \frac{i_{c1(sc)}}{v_{tb1}} \times \frac{v_{te2}}{i_{c1(sc)}} \times \frac{i_{c2(sc)}}{v_{te2}} \times \frac{v_o}{i_{c2(sc)}} \\ &= G_{m1} \times (-r_{ic1} \parallel R_{C1}) \times (-G_{me2}) \times (-r_{ic2} \parallel R_{tc2})\end{aligned}$$

The expressions for  $r_{in}$  and  $r_{out}$  are the same as for the cascode amplifier in Fig. 17(b). The resistance seen looking out of the collector of  $Q_1$  is  $R_{tc1} = R_{C1} \parallel r_{ie2}$ .

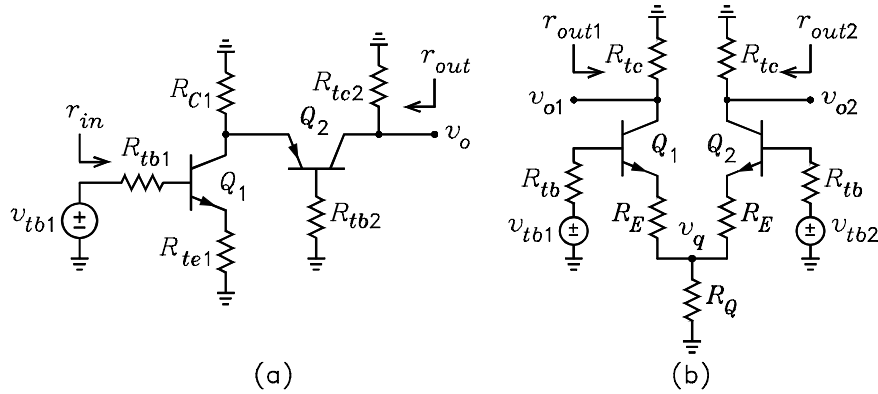


Figure 18: (a) Second cascode amplifier. (b) Differential amplifier.

## The Differential Amplifier

Figure 18(b) shows the ac signal circuit of a differential amplifier. For the case of an active tail bias supply, the resistor  $R_Q$  represents its small-signal ac resistance. We assume that the transistors are identical, biased at the same currents and voltages, and have identical small-signal parameters. Looking out of the emitter of  $Q_1$ , the Thévenin voltage and resistance are given by

$$\begin{aligned}v_{te1} &= v_{e2(oc)} \frac{R_Q}{R_Q + R_E + r_{ie}} \\ &= v_{tb2} \frac{r_0 + R_{tc} / (1 + \beta)}{r'_e + r_0 + R_{tc} / (1 + \beta)} \frac{R_Q}{R_Q + R_E + r_{ie}}\end{aligned}\tag{67}$$

$$R_{te1} = R_E + R_Q \parallel (R_E + r_{ie}) \quad (68)$$

The small-signal collector voltage of  $Q_1$  is given by

$$\begin{aligned} v_{o1} &= -i_{c1(sc)} (r_{ic} \parallel R_{tc}) = -(G_{mb} v_{tb1} - G_{me} v_{te1}) (r_{ic} \parallel R_{tc}) \\ &= -G_{mb} (r_{ic} \parallel R_{tc}) v_{tb1} \\ &\quad + G_{me} \frac{r_0 + R_{tc} / (1 + \beta)}{r'_e + r_0 + R_{tc} / (1 + \beta)} \frac{R_Q}{r_{ie} + R_E + R_Q} v_{tb2} \end{aligned} \quad (69)$$

By symmetry,  $v_{o2}$  is obtained by interchanging the subscripts 1 and 2 in this equation. The small-signal resistance seen looking into either output is

$$r_{out} = R_{tc} \parallel r_{ic} \quad (70)$$

where  $r_{ic}$  calculated from Eq. (??) with  $R_{te} = R_E + R_Q \parallel (R_E + r_{ie})$ . Although not labeled on the circuit, the input resistance seen by both  $v_{tb1}$  and  $v_{tb2}$  is  $r_{in} = r_{ib}$ .

A second solution of the diff amp can be obtained by replacing  $v_{tb1}$  and  $v_{tb2}$  with differential and common-mode components as follows:

$$v_{tb1} = v_{i(cm)} + \frac{v_{i(d)}}{2} \quad (71)$$

$$v_{tb2} = v_{i(cm)} - \frac{v_{i(d)}}{2} \quad (72)$$

where  $v_{i(d)} = v_{tb1} - v_{tb2}$  and  $v_{i(cm)} = (v_{tb1} + v_{tb2}) / 2$ . Superposition of  $v_{i(d)}$  and  $v_{i(cm)}$  can be used to solve for  $v_{o1}$  and  $v_{o2}$ . With  $v_{i(cm)} = 0$ , the effects of  $v_{tb1} = v_{i(d)} / 2$  and  $v_{tb2} = -v_{i(d)} / 2$  are to cause  $v_q = 0$ . Thus the  $v_q$  node can be grounded and the circuit can be divided into two common-emitter stages in which  $R_{te(d)} = R_E$  for each transistor. In this case,  $v_{o1(d)}$  can be written

$$\begin{aligned} v_{o1(d)} &= \frac{i_{c1(sc)}}{v_{tb1(d)}} \times \frac{v_{o1(d)}}{i_{c1(sc)}} v_{tb1(d)} = G_{m(d)} \times (-r_{ic} \parallel R_{tc}) \frac{v_{i(d)}}{2} \\ &= G_{m(d)} \times (-r_{ic} \parallel R_{tc}) \frac{v_{tb1} - v_{tb2}}{2} \end{aligned} \quad (73)$$

By symmetry  $v_{o2(d)} = -v_{o1(d)}$ .

With  $v_{i(d)} = 0$ , the effects of  $v_{tb1} = v_{tb2} = v_{i(cm)}$  are to cause the emitter currents in  $Q_1$  and  $Q_2$  to change by the same amounts. If  $R_Q$  is replaced by two parallel resistors of value  $2R_Q$ , it follows by symmetry that the circuit can be separated into two common-emitter stages each with  $R_{te(cm)} = R_E + 2R_Q$ . In this case,  $v_{o1(cm)}$  can be written

$$\begin{aligned} v_{o1(cm)} &= \frac{i_{c1(sc)}}{v_{tb1(cm)}} \times \frac{v_{o1(cm)}}{i_{c1(sc)}} v_{tb1(cm)} = G_{m(cm)} (-r_{ic} \parallel R_{tc}) v_{i(cm)} \\ &= G_{m(cm)} \times (-r_{ic} \parallel R_{tc}) \frac{v_{tb1} + v_{tb2}}{2} \end{aligned} \quad (74)$$

By symmetry  $v_{o2(cm)} = v_{o1(cm)}$ .

Because  $R_{te}$  is different for the differential and common-mode circuits,  $G_m$  and  $r_{ib}$  are different. However, the total solution  $v_{o1} = v_{o1(d)} + v_{o1(cm)}$  is the same as that given by Eq. (69), and similarly for  $v_{o2}$ . Note that  $r_{ic}$  is the same for both solutions and is calculated with  $R_{te} = R_E + R_Q \parallel (R_E + r_{ie})$ . The small-signal base currents can be written  $i_{b1} = v_{i(cm)} / r_{ib(cm)} + v_{i(d)} / r_{ib(d)}$  and  $i_{b2} = v_{i(cm)} / r_{ib(cm)} - v_{i(d)} / r_{ib(d)}$ . If  $R_Q \rightarrow \infty$ , the common-mode gain is very small, approaching 0 as  $r_0 \rightarrow \infty$ . In this case, the differential solutions can be used for the total solutions. If  $R_Q \gg R_E + r_{ie}$ , the common-mode solutions are often approximated by zero.

## Small-Signal High-Frequency Models

Figure 19 shows the hybrid- $\pi$  and T models for the BJT with the base-emitter capacitance  $c_\pi$  and the base-collector capacitance  $c_\mu$  added. The capacitor  $c_{cs}$  is the collector-substrate capacitance which is present in monolithic integrated-circuit devices but is omitted in discrete devices. These capacitors model charge storage in the device which affects its high-frequency performance. The capacitors are given by

$$c_\pi = c_{je} + \frac{\tau_F I_C}{V_T} \quad (75)$$

$$c_\mu = \frac{c_{jc}}{[1 + V_{CB}/\phi_C]^{m_c}} \quad (76)$$

$$c_{cs} = \frac{c_{jcs}}{[1 + V_{CS}/\phi_C]^{m_c}} \quad (77)$$

where  $I_C$  is the dc collector current,  $V_{CB}$  is the dc collector-base voltage,  $V_{CS}$  is the dc collector-substrate voltage,  $c_{je}$  is the zero-bias junction capacitance of the base-emitter junction,  $\tau_F$  is the forward transit time of the base-emitter junction,  $c_{jc}$  is the zero-bias junction capacitance of the base-collector junction,  $c_{jcs}$  is the zero-bias collector-substrate capacitance,  $\phi_C$  is the built-in potential, and  $m_c$  is the junction exponential factor. For integrated circuit lateral pnp transistors,  $c_{cs}$  is replaced with a capacitor  $c_{bs}$  from base to substrate, i.e. from the B node to ground.

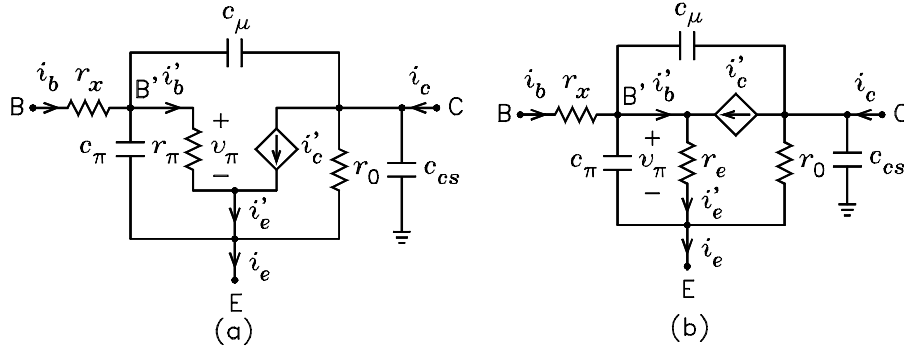


Figure 19: High-frequency small-signal models of the BJT. (a) Hybrid- $\pi$  model. (b) T model.

In these models, the currents are related by

$$i'_c = g_m v_\pi = \beta i'_b = \alpha i'_e \quad (78)$$

These relations are the same as those in Eq. (26) with  $i_b$  replaced with  $i'_b$ .