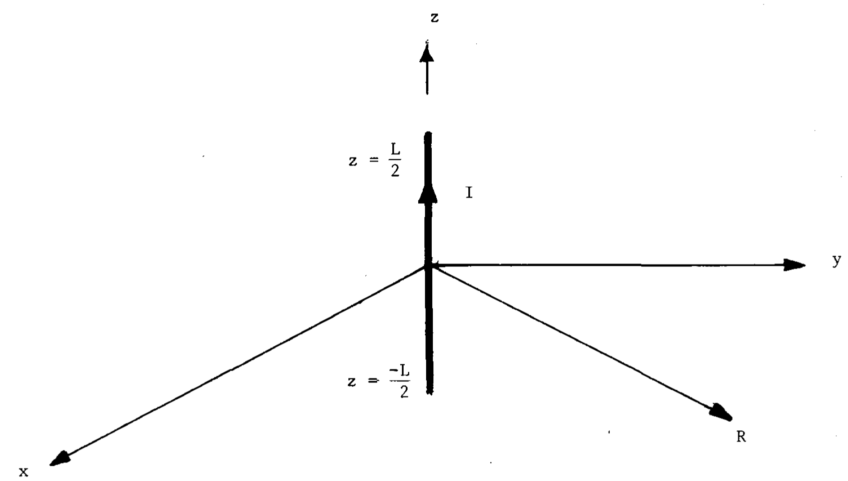


Fig. 1. A wire of length $L$ carrying dc current $I$.

The electric field on the $xy$ plane due to point charges $-It$ at $z = -(L/2)$ and $+It$ at $z = L/2$ can be calculated to be

$$E(\rho) = -\hat{a}_z \frac{It}{4\pi\epsilon_0} \frac{L}{\left[\rho^2 + \left(\frac{L}{2}\right)^2\right]^{3/2}}.$$

From which it follows that

$$D(\rho, t) = -\hat{a}_z \frac{It}{4\pi} \frac{L}{\left[\rho^2 + \left(\frac{L}{2}\right)^2\right]^{3/2}} \tag{7}$$

and

$$\frac{\partial D}{\partial t} = -\hat{a}_z \frac{I}{4\pi} \frac{L}{\left[\rho^2 + \left(\frac{L}{2}\right)^2\right]^{3/2}} \tag{8}$$

Now we can evaluate the displacement current term which is needed in (4)

$$\int_s \frac{\partial D}{\partial t} \cdot ds = \int_{\rho=0}^{\rho} -\hat{a}_z \frac{I}{4\pi} \frac{L}{\left[\rho^2 + \left(\frac{L}{2}\right)^2\right]^{3/2}} \cdot \hat{a}_z \, 2\pi\rho\partial\rho$$

or

$$\int_s \frac{\partial D}{\partial t} \cdot ds = I\left[\frac{L}{\left[\rho^2 + \left(\frac{L}{2}\right)^2\right]^{3/2}} - 1\right] \tag{9}$$

upon substitution of (9) in (6) we obtain the expression for $B$ as

$$B_\phi = \frac{\mu_0 IL}{4\pi\rho\sqrt{\rho^2 + \left(\frac{L}{2}\right)^2}} \tag{10}$$

which is the same result as (5). We have thus shown that the law of Biot–Savart is equivalent to the complete Maxwell's equation (4) in dealing with this example.

## IV. Conclusion

We have shown that the law of Biot–Savart is capable of giving the magnetic flux density due to dc currents in complete circuits or segments of circuits. The charge accumulations at the ends of such segments are automatically accounted for in this law. In such cases the law of Biot–Savart is equivalent to the complete Maxwell's equation given by (4).

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# Determining Spice Parameter Values for BJT's

N. R. MALIK

***Abstract*—This paper briefly compares the roles of hybrid-pi, two-port, and Spice transistor parameters. It then describes a step-by-step procedure for determining numerical parameter values for a dynamic Spice model of a BJT from manufacturer's measured data. The procedure is illustrated with an example.**

## I. Introduction

In the undergraduate electrical engineering curriculum there has been growing use of Spice for simulation of electronic circuits. Spice is now well-established in the first circuits course [1], and it is being included in revisions [2] and in new texts [3] in undergraduate electronics. New products which enable using Spice on

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microcomputers [4], minicomputers, and computer networks [5] are also being rapidly introduced.

In modern design courses, students are often required to use Spice simulations for close support of laboratory design projects. In such courses students design and test electronic circuits using either discrete or integrated components. An important course objective is to teach students to integrate computer simulations with the actual laboratory work. Reasonably accurate Spice simulations are very useful in design iterations before going into the laboratory. They are also helpful in explaining experimental results after there has been an opportunity to measure parameters of specific devices and components which have been used.

Two practical difficulties have surfaced in our attempts to teach such a course. One of these is student confusion over the roles played by the various kinds of transistor parameters. Each parameter class is usually presented in its own context and in isolation from the others. The second is the absence in textbooks of a straightforward procedure for determining Spice parameters for bipolar transistors from published data sheet information. Although some textbooks describe how to determine hybrid-pi parameters from published data [3], none address this relatively new and specific concern of finding the Spice parameters. This paper is an outgrowth of notes written to help students deal with these two problems.

## II. Roles of the Various BJT Parameters

Most familiar to advanced students are the parameters of the high-frequency hybrid-pi transistor model. The values of these parameters, which are functions of the quiescent operating point ($Q$-point), are needed for estimating frequency response by simple hand calculations.

Easiest to measure, and therefore more likely to be available on manufacturer's data sheets, are the small signal $h$-parameters plus certain capacitance values. Since all of these parameters are also functions of operating point, the manufacturer always states the $Q$-point at which the measurements were made. Occasionally other parameters such as $y$-parameters may be given instead of $h$-parameters, depending on the applications of the particular device. In these cases, tables of conversion are available [1] to convert from any other parameters to the $h$-parameters at the same operating point.

A third parameter set is that used in Spice transistor models. These are based upon physical properties of transistors and are *valid at any operating point*. Once Spice parameters are determined, others such as the hybrid-pi parameters can be calculated for any operating point. In fact, after computing the $Q$-point of each BJT, many implementations of Spice compute and print out the values of the hybrid pi parameters.

The next section outlines a step-by-step procedure which can be used to estimate values for all of the Spice parameters required for a linear, dynamic, model valid for forward active operation using commonly-available transistor data sheet information. The procedure makes use of some approximations given in [2] and [6].

## III. Computing Spice Parameters

### Given Information

One usually begins with the following data sheet information:

1) a given point on the $I_E$ versus $V_{BE}$ curve,

2) measured values of the small-signal $h$-parameters; $h_{fe}$, $h_{ie}$, $h_{oe}$, and $h_{re}$, at a given operating point,

3) measured values of collector-base capacitance ($C_\mu = C_{CBO}$), emitter-base capacitance ($C_{je} = C_{EBO}$), and collector-substrate capacitance ($C_{CIO}$) at given operating points,

4) the unity gain frequency $f_\tau$ measured at a given operating point.

(Unfortunately the notation used for these parameters sometimes differs according to the manufacturer.)

The following step-by-step procedure leads to the required Spice parameters, indicated by boldface characters in the equations.

### Static Parameters

1) Compute the "transport saturation current" using

$$\mathbf{IS} = I_E \exp\left[-V_{BE}/V_T\right], \tag{1}$$

where $V_T = \mathrm{k}T/q$.

2) The ideal "maximum forward beta" without correction for Early effect is given by

$$\mathbf{BF} = \beta = h_{fe}. \tag{2}$$

3) Compute the transconductance at the given $h$-parameter operating point using

$$g_m = I_c/V_T. \tag{3}$$

4) Compute $r_\pi$ using

$$r_\pi = \beta \big/ g_m. \tag{4}$$

5) Compute $r_\mu$ from

$$r_\mu = r_\pi / h_{re}. \tag{5}$$

6) Compute $r_o$ using

$$1/r_o = h_{oe} - \beta \big/ r_\mu. \tag{6}$$

7) Compute the "forward Early voltage" using

$$\mathbf{V_{AF}} = r_o I_c, \tag{7}$$

where $I_c$ is the bias current at which the $h$-parameters were measured.

8) Compute the value of the "zero-bias base resistance" using

$$\mathbf{RB} = r_x = h_{ie} - r_\pi. \tag{8}$$

(Because of measurement errors this value occasionally turns out to be negative and must be replaced with an estimate.)

### Dynamic Model Parameters

The following procedure gives the remaining Spice parameters required for a dynamic forward-active model.

9) Use

$$C_\mu = \mathbf{CJC}/\left[1 + (V_{CB}/\mathrm{VJC})\right]^{\mathrm{MJC}} \tag{9}$$

to find the "base-collector zero-bias depletion capacitance." In this equation, the value of $C_\mu$ will be given as well as the voltage, $V_{CB}$, at which the measurement was made. Use VJC = 0.55 and MJC = 0.5 for I.C.s. [6] Solve for **CJC**. (Alternative notations sometimes used for $C_\mu$ on data sheets are $C_c$, $C_{ob}$, $C_{cb}$, and $C_{CBO}$.)

10) Use

$$C_{je} = \mathbf{CJE}/\left[1 + (V_{EB}/\mathrm{VJE})\right]^{\mathrm{MJE}} \tag{10}$$

to find the "depletion" part of $C_\pi$; that is, the "base-emitter zero-bias depletion capacitance." In this equation, $V_{EB}$ is the voltage at which the measurement was made. Use VJE = 0.7 and MJE = 0.33 for I.C.s. [6] Solve for **CJE**.

11) Use

$$C_{CIO} = \mathbf{CJS}/\left[1 + (V_{CI}/\mathrm{VJS})\right]^{\mathrm{MJS}} \tag{11}$$

to find the "collector-substrate zero-bias depletion capacitance," where $V_{CI}$ is the voltage at which $C_{CIO}$ was measured. Use MJS = 0.5 and VJS = 0.52 for I.C.s. [6] Solve for **CJS**. This step is omitted for discrete transistors.

12) Use the following procedure to find the "forward transit time," **TF**.

i) Compute $g_m$ at the $Q$-point where the unity gain frequency $f_\tau$ was measured using (3).

ii) Compute $C_\mu$ at the $Q$-point where the unity gain frequency $f_\tau$ was measured using (9).

iii) Compute

$$C_\pi = (g_m/2\pi f_\tau) - C_\mu. \qquad (12)$$

iv) Estimate $C_{je}$, the part of $C_\pi$ attributed to depletion capacitance, at the $Q$-point where the unity gain frequency was measured. For strongly forward-biased pn junctions, (10) does not give correct results [6]. A suggested estimate [6, p. 40] is

$$C_{je} = 2 \times \mathbf{CJE}. \qquad (13)$$

v) Compute the "charge-storage" part of $C_\pi$, $C_b$, using

$$C_b = C_\pi - C_{je}. \qquad (14)$$

vi) The forward transit time is now found from

$$\mathbf{TF} = C_b/C_\pi. \qquad (15)$$

## IV. Example

Spice parameters for the BJT's of the CA3086, a general purpose NPN transistor array, were obtained as follows.

1) Curves of $V_{BE}$ versus temperature were given for several values of $I_E$, from which the point $(I_E, V_{BE}) = (0.5\text{ mA}, 0.68\text{ V})$ was obtained.

From Eq. (1), $\mathbf{IS} = 7.69 \times 10^{-16}$ A.

2) Given hybrid parameters, measured at $I_C = 1$ mA and $V_{CE} = 3$ V, were

$$h_{fe} = 100 \quad h_{ie} = 3.5\text{ K}\Omega \quad h_{oe} = 15.6\ \mu s \quad h_{re} = 1.8 \times 10^{-4}$$

From (2), $\mathbf{BF} = 100$.
From (3), $g_m = 10^{-3}/0.025 = 0.040$ S.
From (4), $r_\pi = 100/0.040 = 2.5$ KΩ.
From (5), $r_\mu = 2.5\text{ K}/1.84 \times 10^{-4} = 13.9$ MΩ.
From (6), $1/r_o = 15.6 \times 10^{-6} - 100/13.9 \times 10^6$, therefore $r_o = 1.19 \times 10^5\ \Omega$.
From (7), $\mathbf{VAF} = (1.190 \times 10^5) \times 1 \times 10^{-3} = 119$ V.
From (8), $\mathbf{RB} = 3.5\text{ K} - 2.5\text{ K} = 1$ KΩ.

3) The following capacitance values and measurement conditions were listed on the data sheet.

$$C_{CBO} = 0.58\text{ pF, at } V_{CB} = 3\text{ V}$$

$$C_{EBO} = 0.60\text{ pF, at } V_{EB} = 3\text{ V}$$

$$C_{CI} = 2.8\text{ pF pF, at } V_{CI} = 3\text{ V}$$

From (9), $\mathbf{CJC} = 0.58 \times 10^{-12}[1 + 3/0.55]^{0.5}$.
From (10), $\mathbf{CJE} = 0.60 \times 10^{-12}[1 + 3/0.7]^{0.33}$
From (11), $\mathbf{CJS} = 2.8 \times 10^{-12}[1 + 3/0.52]^{0.5}$

4) The unity gain frequency and its measurement conditions were

$$f_\tau = 550\text{ MHz. at } I_C = 3\text{ mA and } V_{CE} = 3\text{ V}.$$

Computing the value of $C_\mu = C_{CBO}$ at the $f_\tau$ operating point gives

$$C_\mu = 1.474 \times 10^{-12}/[1 + 2.3/0.55]^{0.5} = 0.648\text{ pF}.$$

From (12), $C_\pi = (0.120/2\pi 550 \times 10^6) - 0.648 \times 10^{-12} = 34.08 \times 10^{-12}$ Fd.
From (13), $C_{je} = 2 \times \mathbf{CJE} = 2.078 \times 10^{-12}$ Fd.
From (14), $C_b = 34.08 \times 10^{-12} - 2.078 \times 10^{-12} = 32.00 \times 10^{-12}$ Fd.
From (15), $\mathbf{TF} = 32 \times 10^{-12}/0.120 = 2.667 \times 10^{-9}$ s.

Simulation results obtained using these parameter values agreed quite well with experimental results obtained in the laboratory.

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# The Discrete Fourier Transform Data Sequence Need Not Be Circularly Defined

A. VAN DEN BOS

*Abstract*—**In literature the finite discrete Fourier transform (DFT) data sequence is usually assumed to be circular. It is shown that the familiar DFT theorems can be proved without this often somewhat artificial assumption.**

## I. Introduction

In the literature on the finite discrete Fourier transform (DFT) various assumptions are found with respect to the data sequence itself and/or the hypothetical sequences preceding and following it. Cooley, Lewis, and Welch [1], Oran Brigham [2], and Kay and Marple [3] assume that the finite data sequence is one period of an otherwise infinite periodic sequence. Oppenheim and Schafer [4] also represent the data sequence as one period of a periodic sequence. However, outside this period the amplitudes are assumed to be equal to zero. Moreover, the shifted version of the data sequence is represented as one period of the equally shifted periodic sequence. The purpose of this paper is not to question the correctness or the usefulness of these points of view. The purpose is to investigate whether or not it is possible to make no assumptions at all about the data sequence and the sequences preceding and following it. The motivation for this is that the DFT data representations described above may be puzzling for the student or user of the DFT. The data sequence available will often clearly not be one period of a periodic sequence nor will the sequences preceding and following it be zero-valued. For that purpose, in the next section three key DFT theorems (inversion, shift, and convolution) will be reconsidered without assumptions on the data sequence. The resulting conclusions are summarized in a final section.

## II. Reconsideration of Three Key DFT Theorems

Let $x(n)$, $n = 0, \cdots, N-1$ be an otherwise unspecified and possibly complex data sequence. Define

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \qquad k = 0, \cdots, N-1 \qquad (1)$$

as the discrete Fourier transform (DFT) of $x(n)$, $n = 0, \cdots, N-1$ where $W_N = \exp(-j2\pi/N)$ with $j = \sqrt{-1}$. Then the *inversion theorem* states that the inverse discrete Fourier transform (IDFT) defined by

$$\frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-nk} \qquad n = 0, \cdots, N-1 \qquad (2)$$

is equal to $x(n)$, $n = 0, \cdots, N-1$. The proof of this theorem does not require $x(n)$ to be periodic, circular, or equal to zero outside $n = 0, \cdots, N-1$; see [5].

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