Physical Definition of Saturation Current

Recall the following:

$$J_n = q\mu_n n(x)E(x) + qD_n \frac{dn(x)}{dx}$$

$$J_p = q\mu_p p(x)E(x) - qD_p \frac{dp(x)}{dx}$$

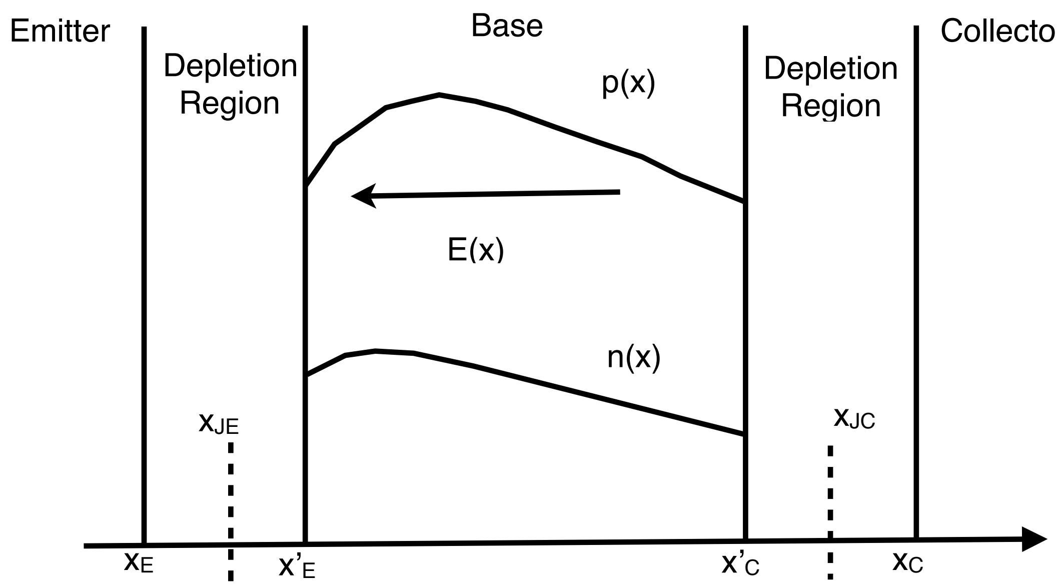
If the hole current is assumed to be zero, then

$$J_p = q\mu_p p(x)E(x) - qD_p \frac{dp(x)}{dx} = 0$$
$$\therefore$$
$$E(x) = \frac{D_p}{\mu_p}\frac{1}{p(x)}\frac{dp(x)}{dx}$$

$$J_n = q\mu_n \left[\frac{D_p}{\mu_p}\frac{1}{p(x)}\frac{dp(x)}{dx}\right]n(x) + qD_n \frac{dn(x)}{dx}$$

$$p(x)J_n = qD_n n(x)\frac{dp(x)}{dx} + qD_n p(x)\frac{dn(x)}{dx}$$
$$= qD_n\left[n(x)\frac{dp(x)}{dx} + p(x)\frac{dn(x)}{dx}\right] = qD_n \frac{d}{dx}\left[n(x)p(x)\right]$$

We know integrate the above equation from x_E to x_C , where x_E is the position of the emitter side of the emitter base depletion region and x_C is the position of the collector side of the emitter base depletion region, as illustrated below:



Since current density J_n is constant for dc and is independent of x , assuming negligible recombination in the base region (recombination in the depletion region is independently modelled by the two non-ideal of Gummel Poon Static Model, so it is assumed to be zero here), J_n is taken out of the integral. Then

$$J_n \int_{x_E}^{x_C} p(x)dx = qD_n \int_{x_E}^{x_C} \frac{d}{dx}\left[n(x)p(x)\right]dx = qD_n\left[n(x_C)p(x_C) - n(x_E)p(x_E)\right]$$

$$J_n = \frac{qD_n\left[n(x_C)p(x_C) - n(x_E)p(x_E)\right]}{\int_{x_E}^{x_C} p(x)dx}$$

Now we apply the depletion approximation. It assumes that there are no or negligible mobile carriers in a depletion region - this means that the field experienced by the carriers and the thickness of the depletion region are such that the carriers are transported instantly across it.

We apply the depletion approximation to the limits of the integral replaced by x_E' and x_C' with the integration performed in the neutral base region only. The change in the integration limits results from the assumption that $p(x)$ is approximately zero inside the depletion regions.

$$\therefore$$

$$I_n \simeq -\frac{qD_n A_j n_i^2\left[\left(e^{qV_{BE}/kT} - 1\right) - \left(e^{qV_{BC}/kT} - 1\right)\right]}{\int_{x_E'}^{x_C'} p(x)dx}$$

where A_j is the one dimensional cross sectional area and I_n is the total dc minority current in the positive *x* direction that results from minority carriers injected into the base at the emitter and/or the collector. It is represented in the Gummel-Poon Static model by the current generator I_{CT} .

Recall that we had previously defined an expression for I_{CT} during our analysis of Eber-Moll Model:

$$I_{CT} = I_S\left[\left(e^{qV_{BE}/kT} - 1\right) - \left(e^{qV_{BC}/kT} - 1\right)\right]$$

A direct comparison between these two equations yields the physical definition of I_S . Eber-Moll considers I_S a fundamental constant of the device. However, the above equation, I_n that is, implies that under high level injection, $p(x)$ is a function of the applied bias. To reconcile this definition, I_{ss} is introduced in the Gummel-Poon model

At low level injection, the combination of I_n and I_{CT} gives the following:

$$I_{CT}\Big|_{\text{low level}} = \frac{qD_n n_i^2 A_j}{\int_{x'_E}^{x'_C} N_A(x)dx}\left[\left(e^{qV_{BE}/kT}-1\right)-\left(e^{qV_{BC}/kT}-1\right)\right]$$

$p(x)$ has been replaced by $N_A(x)$ since at low current levels in the neutral base region, $p(x)$ approximates to $N_A(x)$.

The fundamental constant I_{SS} is defined at zero V_{BE} and V_{BC} is given as:

$$I_{SS} = \frac{qD_n n_i^2 A_j}{\int_{x'_{E0}}^{x'_{C0}} N_A(x)dx}$$

The limits of the integral are the values of x'_E and x'_C when the applied junction voltages are zero.

Concept of the Total Majority Charge

If we multiply $\int_{x'_E}^{x'_C} p(x)dx$ by q and A_j, the result represents the total majority charge in the neutral base region and this is represented by Q_B :

$$Q_B = \int_{x'_E}^{x'_C} qA_j p(x)dx$$

The zero bias majority base charge, Q_{B0} is defined as:

$$\int_{x'_{E0}}^{x'_{C0}} qA_j N_A(x)dx$$

The normalised majority base charge, q_b is defined as:

$$q_b = \frac{Q_B}{Q_{B0}}$$

If we multiply and divide I_n by Q_{B0} ; and replace I_n with $-I_{CT}$, then we get:

$$I_{CT} = \frac{qD_n n_i^2 A_j}{\int_{x'_E}^{x'_C} p(x)dx}\,\frac{qA_j\int_{x'_{E0}}^{x'_{C0}} N_A(x)dx}{qA_j\int_{x'_{E0}}^{x'_{C0}} N_A(x)dx}\left[\left(e^{qV_{BE}/kT}-1\right)-\left(e^{qV_{BC}/kT}-1\right)\right] = \frac{I_{ss}}{q_b}\left[\left(e^{qV_{BE}/kT}-1\right)-\left(e^{qV_{BC}/kT}-1\right)\right]$$

This new equation was introduced by the Gummel-Poon Model. In Eber-Moll, I_s was assumed constant, whereas in Gummel-Poon, I_s is replaced by I_{ss}/q_b , where I_{ss} is the fundamental constant (defined at zero bias conditions) and q_b is a variable that still needs to be determined.

Solution for q_b

$$q_b = \frac{Q_B}{Q_{B0}} = 1 + \frac{C_{JE}}{Q_{B0}} V_{BE} + \frac{C_{JC}}{Q_{B0}} V_{BC} + \frac{\tau_{BF}}{Q_{BO}} I_{CC} + \frac{\tau_{BR}}{Q_{B0}} I_{EC}$$

$$= 1 + \underbrace{\frac{V_{BE}}{V_A}}_{q_e} + \underbrace{\frac{V_{BC}}{V_A}}_{q_c} + \underbrace{\frac{\tau_{BF}}{Q_{B0}} I_{SS} \frac{e^{qV_{BE}/kT} - 1}{q_b}}_{q_{bF}} + \underbrace{\frac{\tau_{BR}}{Q_{B0}} I_{SS} \frac{e^{qV_{BC}/kT} - 1}{q_b}}_{qbR}$$

Note that both q_e and q_c models the effects of the base width modulation, while q_{bF} and q_{bR} models the effects of high level injection.

We can simplify the equation by applying the following mathematical relation:

$$q_b = 1 + q_e + q_c + \frac{q_2}{q_b} = q_1 + \frac{q_2}{q_b}$$

This is a quadratic equation of the form:

$$q_b^2 - q_b q_1 - q_2 = 0$$

$$\therefore\ q_b = \frac{q_1}{2} + \sqrt{\left(\frac{q_1}{2}\right)^2 + q_2}$$

It therefore follows that

$q_b \simeq q_1$ if $q_2 \ll \frac{q_1^2}{4}$ (low level injection)

$q_b \simeq \sqrt{q_2}$ if $q_2 \gg \frac{q_1^2}{4}$ (high level injection)