Semiconductor Physics

Lecture 3

Intrinsic carrier density

Intrinsic carrier density

$$np = n_i^2 = N_c N_v e^{-\left(E_c - E_v\right)} = N_c N_v e^{\frac{-E_g}{kT}}$$

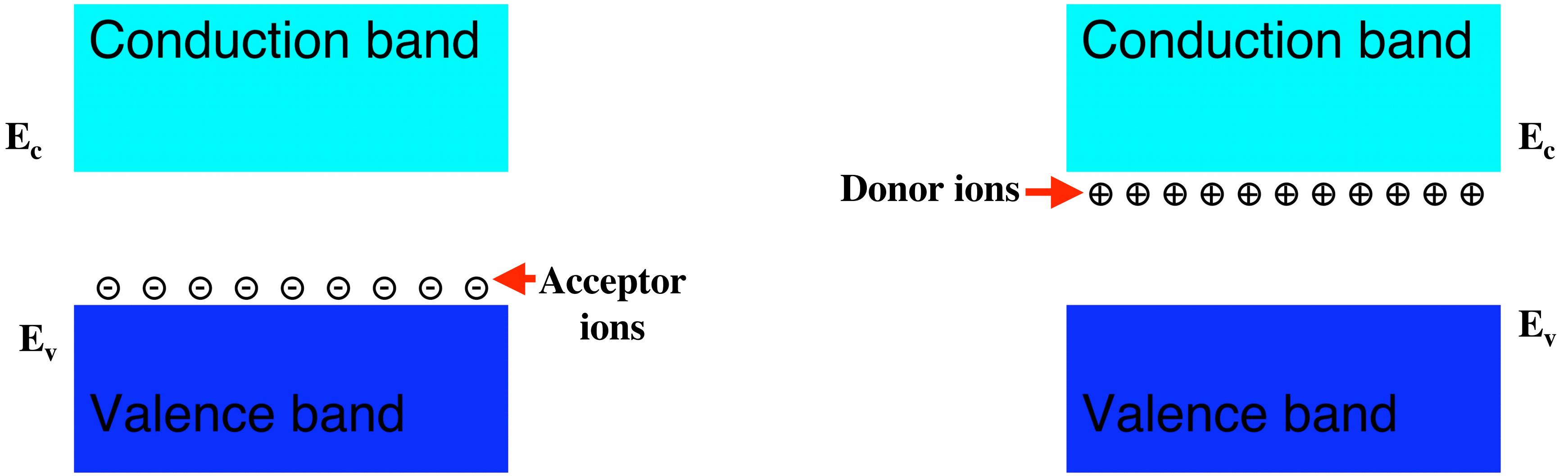
$$n_i = \sqrt{N_c N_v} exp^{\frac{-E_g}{2kT}}$$

Law of mass action

Valid also if we add an impurity which either donates extra electrons or holes the number of carriers remains constant for a given temperature

Doping

Donor levels lie just below conduction band and acceptor levels just above the valence band



Conduction band

E_c

⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖

Acceptor ions

E_v

Valence band

Conduction band

E_c

Donor ions

⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕ ⊕

E_v

Valence band

Donor levels lie E_D below the bottom of the conduction band and acceptor levels E_A above the valence band

Since E_a and E_d are smaller than kT at room temperature the dopant sites are fully ionised so the number of majority carrier (those provided by doping, electrons in the case of n-type, holes in the case of p-type) is simply the number of dopant atoms

Electron and hole concentrations

Recalling the expression for the Fermi energy

$$n_n = \frac{1}{2}(N_D - N_A + \sqrt{(N_D - N_A)^2 + 4n_i^2} \text{ and } p_n = \frac{n_i}{n_n}$$

And the law of mass action

$$n_i = \sqrt{N_c N_v} exp^{\frac{-E_g}{2kT}}$$

Temperature dependence II

T(K)

Fig. 22 Fermi level for Si and GaAs as a function of temperature and impurity concentration. The dependence of the bandgap on temperature is shown in the figure.[6]

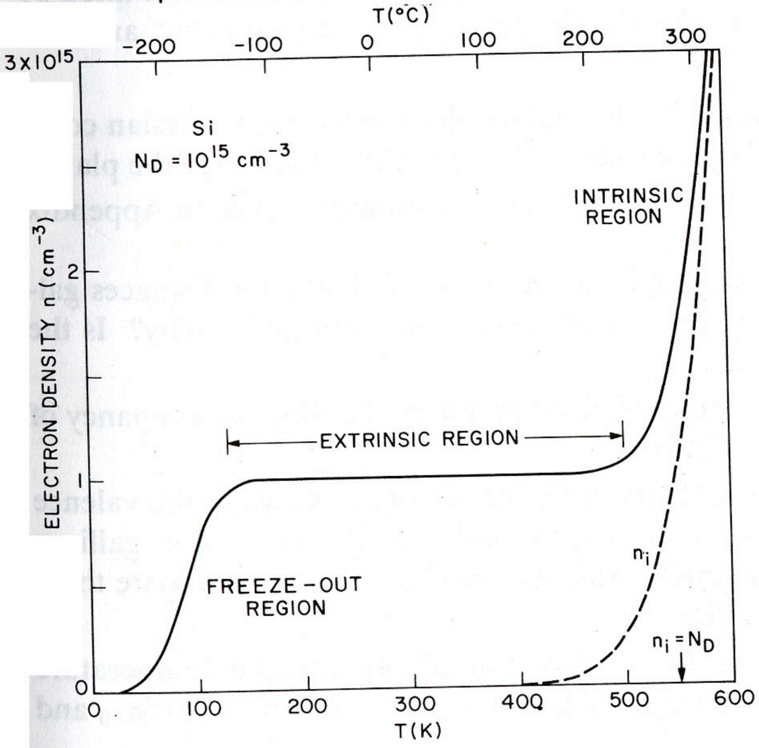


Fig. 23 Electron density as a function of temperature for a Si sample with a donor concentration of 10^{15} cm^{-3}.

Initial rise as dopant sites become fully ionised

Flat region where majority of carriers are from the dopant (good for devices)

Final rise where intrinsic behaviour dominates

Conduction

If we apply an electric field electrons and holes accelerate until they are scattered

We can define an average drift velocity v in terms of the mean time between scattering events

$$-qE\tau = mv$$

This is a completely classical description but we can bring in quantum effects by replacing the electron mass with the effective mass

$$-qE\tau = m^{*}v$$

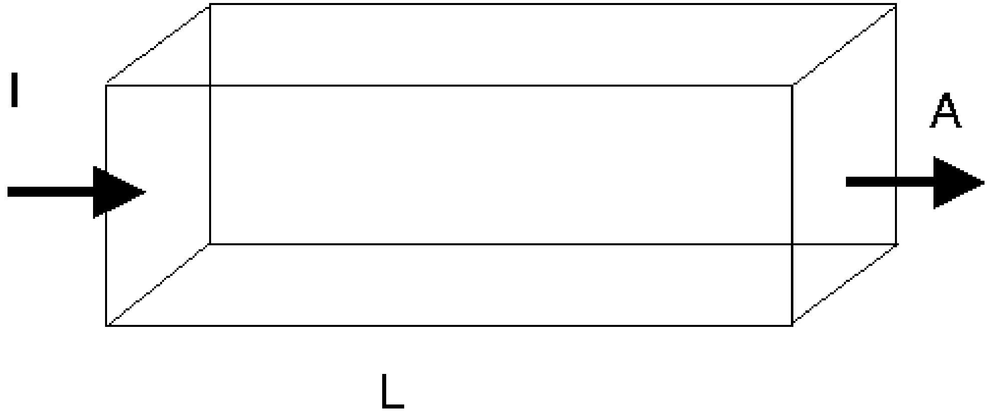
Current density J is then

$$J = nqv = \frac{nq^{2}E\tau}{m^{*}}$$

Conductivity is given by

$$\sigma = \frac{J}{E} = \frac{nq^{2}\tau}{m^{*}}$$

Conductivity



For a metal

$$\sigma = \frac{ne^2\tau}{m} = ne\frac{e\tau}{m} = ne\mu$$

Recognise the second part of the RHS is the mobility

For semiconductors we have more than one type of carrier with more than one mobility electrons and holes have different effective masses and may have different lifetimes

$$\sigma = ne\mu_n + pe\mu_p$$

$$\rho = \frac{1}{\sigma} = \frac{1}{ne\mu_n + pe\mu_p}$$

Conductivity II

$$\sigma = ne\mu_n + pe\mu_p$$

Reason for separating the conductivity into mobility and carrier density

- **In semiconductors both carrier density and mobility are temperature dependent in metals only the mobility is**
- **We have generally at least two types of carrier and we may have even more**

Mobility

The mobile carriers in a semiconductor have an average of 3/2 kT of thermal energy

$$\frac{1}{2}mv_{th}^2 = \frac{3}{2}kT$$

$V_{th}=10^7$ cm/s

they undergo regular collisions with defects and phonons mean free path of 10^{-5} cm lifetime of 1 ps

If an electric field is applied then the carriers will begin to accelerate until they are scattered

$$-eE\tau_c = m_n v_n \text{ or } v_n = -\left(\frac{e\tau_c}{m_n}\right)$$

$$\text{Mobility } \mu_n \equiv -\left(\frac{e\tau_c}{m_n}\right)$$

$$\frac{1}{\tau_c} = \frac{1}{\tau_{lattice}} + \frac{1}{\tau_{impurity}} \quad \frac{1}{\mu_c} = \frac{1}{\mu_{lattice}} + \frac{1}{\mu_{impurity}}$$

Various contributions

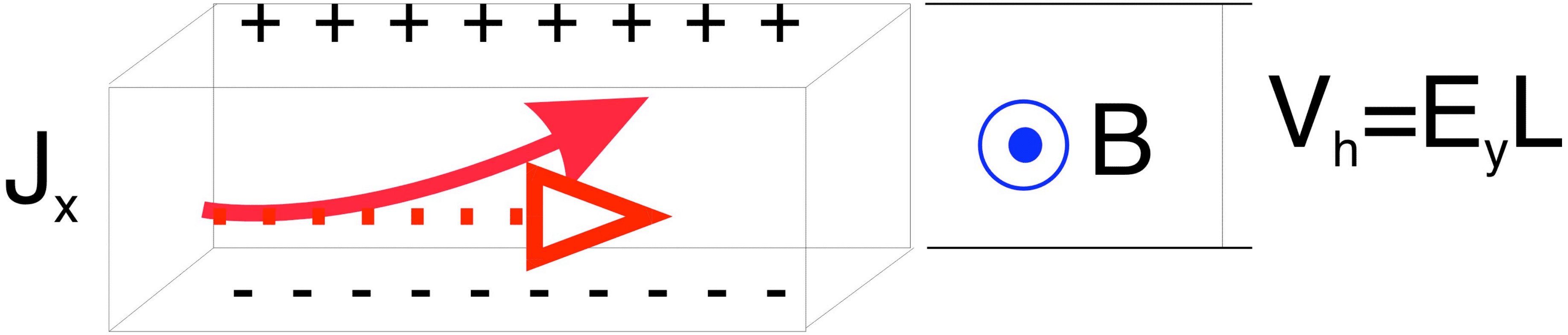
$$\frac{1}{\mu_c} = \frac{1}{\mu_{lattice}} + \frac{1}{\mu_{impurity}}$$

Lattice scattering gives mobility a $T^{-3/2}$ dependence (more phonons at higher T so less mobility)

Impurity scattering has a $T^{3/2}$ effect carriers spend less time near to impurity sites

Also principal impurity sites are the dopant atoms so the more carriers we put in by increasing the doping then the less mobile these carriers are

Hall Effect



For a conductor passing a current in a magnetic field there is a force on the moving current perpendicular to both the current and the magnetic field directions.

Since the carriers cannot escape from the wire a voltage develops which counteracts this force — the Hall voltage

For a single carrier type

$$F_y = qE_y + qv_eB = 0 \text{ where } v_e = \text{electron drift velocity}$$

$$v_e = \mu_e E_x$$

$$E_y = \mu_e B E_x$$

Hall Effect II

But by definition the hall coefficient R_H is

$$R_H \equiv \frac{E_y}{j_x B} \text{ where } j_x = nq\mu_e E_x$$

Substitution for E_y and J_x

$$E_y = \mu_e B E_x$$

$$R_H = \frac{\cancel{\mu_e}\cancel{B}\cancel{E_x}}{nq\cancel{\mu_e}\cancel{E_x}\cancel{B}}$$

Cancel through and have

$$R_H = \frac{1}{nq}$$

A simple way to measure the carrier density and type since q includes the sign of the carriers (-ve for electrons +ve for holes) ?

Hall Effect in Semiconductors

Now have to consider two types of carrier consider the y component of the current density j_y

$$j_{ye} = nq_e(\mu_e E_y + \mu_e^2 B E_x) \text{ and } j_{yh} = nq_h(\mu_h E_y + \mu_h^2 B E_x)$$

So

$$j_y = (nq_e\mu_e + pq_h\mu_h)E_y + (nq_e\mu_e^2 + pq_h\mu_h^2)BE_x$$

But $$E_x = \frac{j_x}{nq_e\mu_e + pq_h\mu_h}$$

$$j_y = (nq_e\mu_e + pq_h\mu_h)E_y + (nq_e\mu_e^2 + pq_h\mu_h^2)B\left(\frac{j_x}{nq_e\mu_e + pq_h\mu_h}\right)$$

Setting $j_y=0$

$$E_y = (nq_e\mu_e^2 + pq_h\mu_h^2)B\left(\frac{j_x}{(nq_e\mu_e + pq_h\mu_h)^2}\right) \qquad R_H \equiv \frac{E_y}{j_x B}$$

Hall Effect in Semiconductors II

So R_H is

$$R_H = \frac{(nq_e\mu_e^2 + pq_h\mu_h^2)}{(nq_e\mu_e + pq_h\mu_h)^2}$$

$$R_H = \frac{n\mu_e^2 - p\mu_h^2}{e(n\mu_e + p\mu_h)^2}$$

Hall Effect appears to depend on mobility (counter intuitive)

Might expect

$$R_H = \frac{1}{e(p-n)}$$

$$R_H = \frac{1}{nq}$$

This is what Sze quotes as the Hall coefficient

The general case and limiting cases

Our model considered only the carrier velocity produced by the applied electric field

In general the carriers have a thermal velocity and will tend to spiral about the magnetic field our model works only if

$$\tau \ll \omega_c = \frac{eB}{m}$$

In general

$$R_H = \frac{\frac{(nq_e\mu_e)^2}{pq_h} + \frac{(pq_h\mu_h)^2}{nq_e} + \left(\frac{1}{npq_eq_h}\right)\left(\frac{1}{pq_h} + \frac{1}{nq_e}\right)B^2}{(nq_e\mu_e + pq_h\mu_h)^2 + \left(\frac{1}{pq_h} + \frac{1}{nq_e}\right)B^2}$$

For small B the terms in B^2 go to zero and we recover our original result

$$R_H = \frac{n\mu_e^2 - p\mu_h^2}{e(n\mu_e + p\mu_h)^2}$$

For large B the B^2 terms dominate and we find the naïve simple view of the Hall coefficient holds

$$R_H = \frac{1}{e(p-n)}$$

Hall effect in intrinsic semiconductors

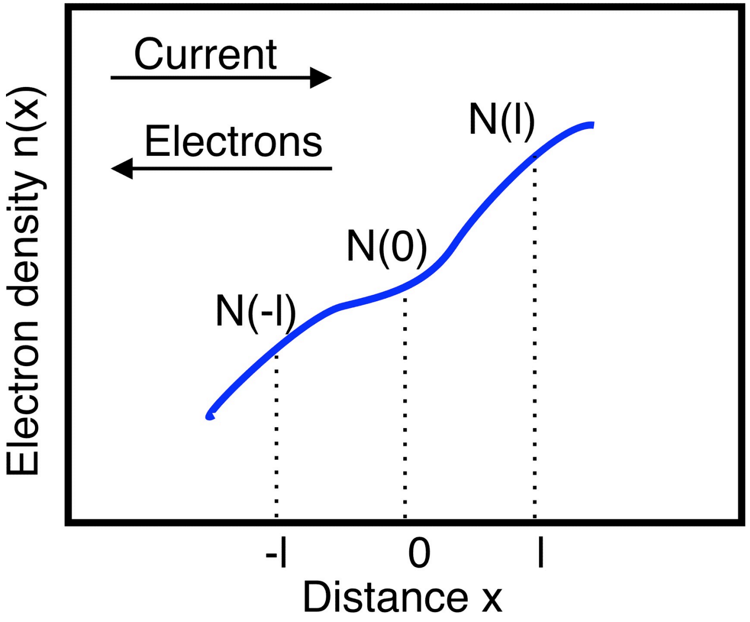
Implies that in general intrinsic semiconductors where n=p will have a Hall effect

$$R_H = \frac{n\mu_e^2 - p\mu_h^2}{e(n\mu_e + p\mu_h)^2}$$

But at high fields this will go to

$$R_H = \frac{1}{e(p-n)}$$

Carrier Diffusion



Up to now we have considered equilibrium thermal properties and the effects of external fields now turn to effects of concentration gradients

Imagine a semiconductor whose electron density varies spatially

The electron mean free path=l

Where $l = v_{th}\tau$

Flow of electrons across a plane at x=0 from left

$$F_1 = \frac{1}{2} n(-l) v_{th}$$

Flow of electrons across a plane at x=0 from right

$$F_2 = \frac{1}{2} n(l) v_{th}$$

Carrier Diffusion II

Net flow

$$F = F_1 - F_2 = \frac{1}{2}n(-l)v_{th} - \frac{1}{2}n(l)v_{th}$$

Replace n(-l) and n(l) by taylor expansion about n(0)

$$F = \frac{1}{2}v_{th}((n(0) - l\frac{dn}{dx}) - (n(0) + l\frac{dn}{dx}))$$

$$F = -v_{th}l\frac{dn}{dx} \equiv -D_n\frac{dn}{dx} \text{ where } D_n \equiv v_{th}l \text{ is the diffusivity}$$

Each carrier carries a charge and so carrier diffusion gives rise to a current

$$j_n = q_nD_n\frac{dn}{dx}$$

Einstein relation

Equipartition for a 1-D case implies

$$\frac{1}{2}m_n v_{th}^2 = \frac{1}{2}kT$$

and given

$$\mu_n = \frac{q_n \tau_c}{m_n} = \frac{q_n l}{m_n v_{th}}$$

$$j_n = q_n D_n \frac{dn}{dx}$$

$$D_n \equiv -v_{th} l$$

$$D_n = (\frac{kT}{q})\mu_n$$

Einstein relationship relating two transport properties

Diffusivity (freedom of electrons to move under concentration gradients)

and mobility (freedom of electrons to move under electric field)

Current density equations

In general we have currents flowing because of both concentration gradients and electric fields

For electrons

$$j_n = q\mu_n nE + qD_n \frac{dn}{dx}$$

For holes

$$j_p = q\mu_p nE - qD_p \frac{dp}{dx}$$

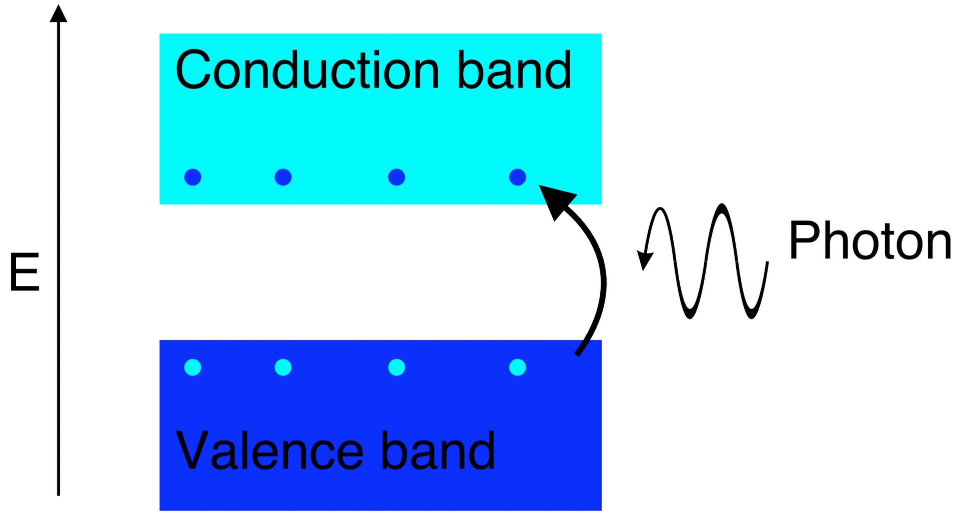
The total conductivity

$$j_{cond} = j_p + j_n$$

Carrier injection

So far we have considered cases where the semiconductor is in thermal equilibrium (at least locally) and the law of mass action holds

$$np = n_i^2 = N_c N_v e^{-(E_c - E_v)} = N_c N_v e^{\frac{-E_g}{kT}}$$



Non-equilibrium case

We can inject extra carrier by various methods

- by shining light on the material
- by biasing a pn junction

Magnitude of the number of carriers determines the level of injection

Carrier injection by light

Since a photon creates an electron hole pair $\Delta p = \Delta n$

Case of n-doped silicon where $N_D = 10^{15}$ cm^{-3} and $n_i = 10^{10}$ cm^{-3} $n = 10^{15}$ cm^{-3} , $p = 10^5$ cm^{-3}

If we inject $\Delta p = \Delta n = 10^{12}$ cm^{-3}

Increase p by 7 orders of magnitude

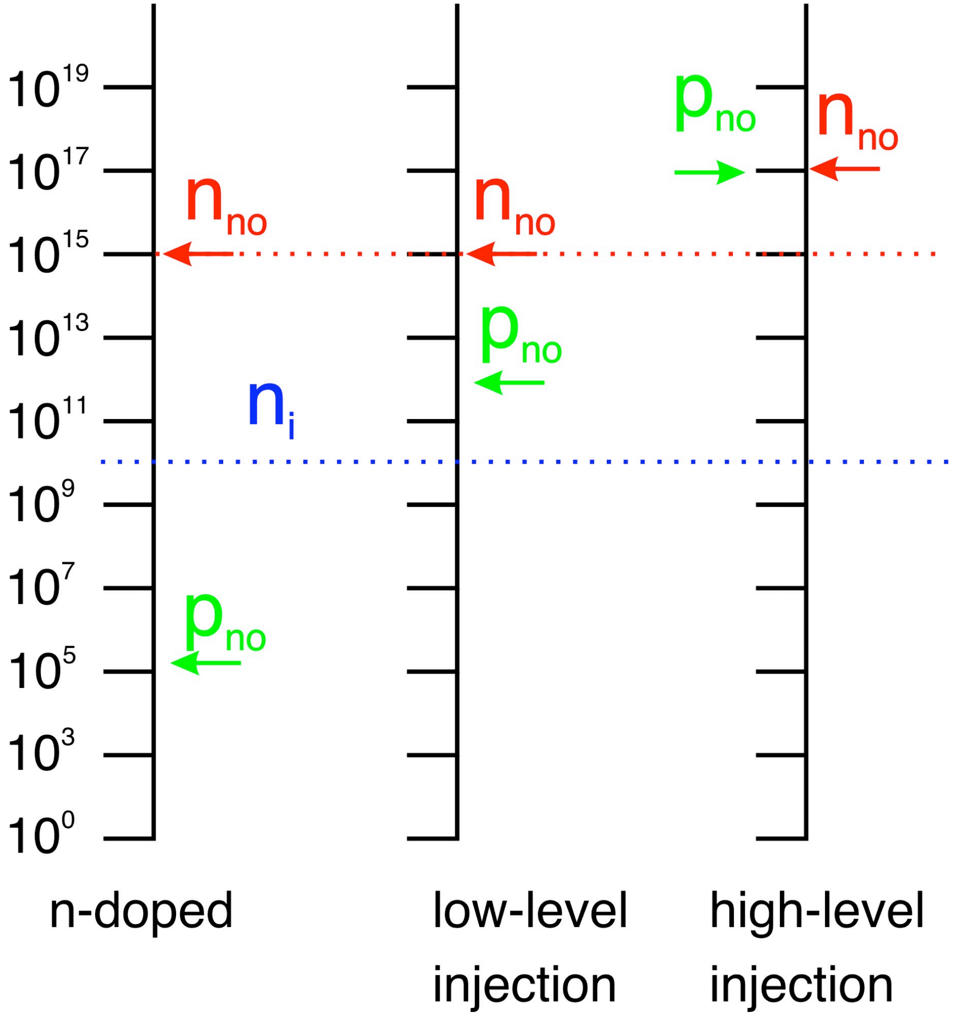
Increase n by 1%

Low level injection affects only minority carrier concentration

If $\Delta p = \Delta n = 10^{17}$ cm^{-3}

Overwhelm the equilibrium majority carrier concentration

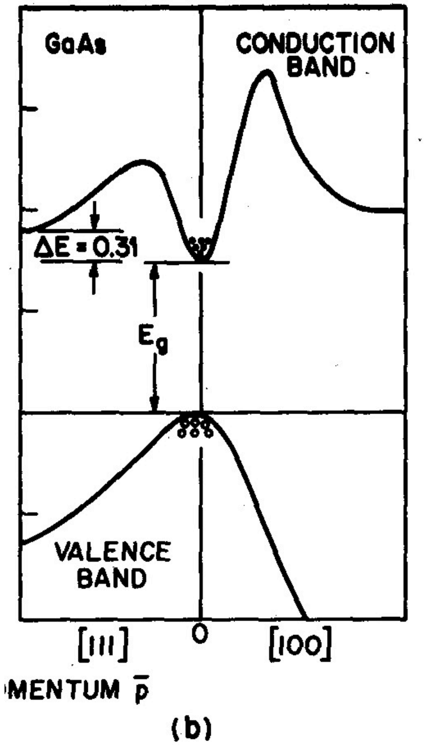
High level injection

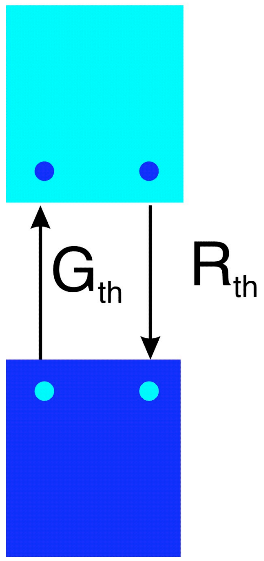


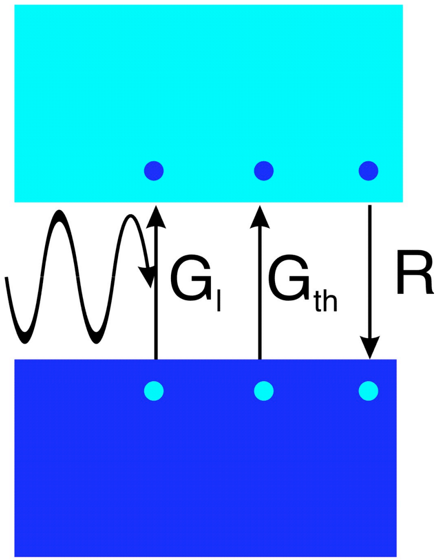
Direct recombination

Injected carriers are a non-equilibrium phenomenon and are removed bv
recombination of electron hole pairs

In a direct gap (like GaAs semiconductor this occurs directly electrons
and holes simply combine and annihilate
(sometimes with the emission of a photon)







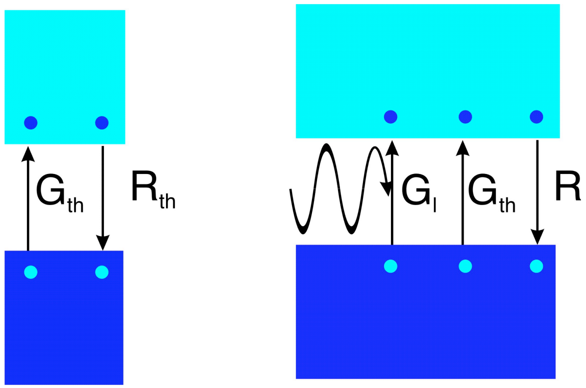
Recombination rate is proportional to the concentration of holes and electrons

$$R = \beta n p$$

For thermal equilibrium case where carrier concentrations are constant

$$G_{th} = R_{th} = \beta n_{no} p_{no}$$

Direct recombination II



Rates of recombinatioon and generation are thus

$$R = \beta(n_{no} + \Delta n)(p_{no} + \Delta p)$$
$$G = G_L + G_{th}$$

Rate of change of hole concentration is given by

$$\frac{\partial p_n}{\partial t} = G - R = G_L + G_{th} - R$$

For steady state define U the net recombination rate U=R-G_{th}=G_l

$$G_{th} = R_{th} = \beta n_{no} p_{no} \qquad R = \beta(n_{no} + \Delta n)(p_{no} + \Delta p)$$

Recalling Δn= Δp

$$U = \beta(n_{no} + p_{no} + \Delta p)\Delta p$$

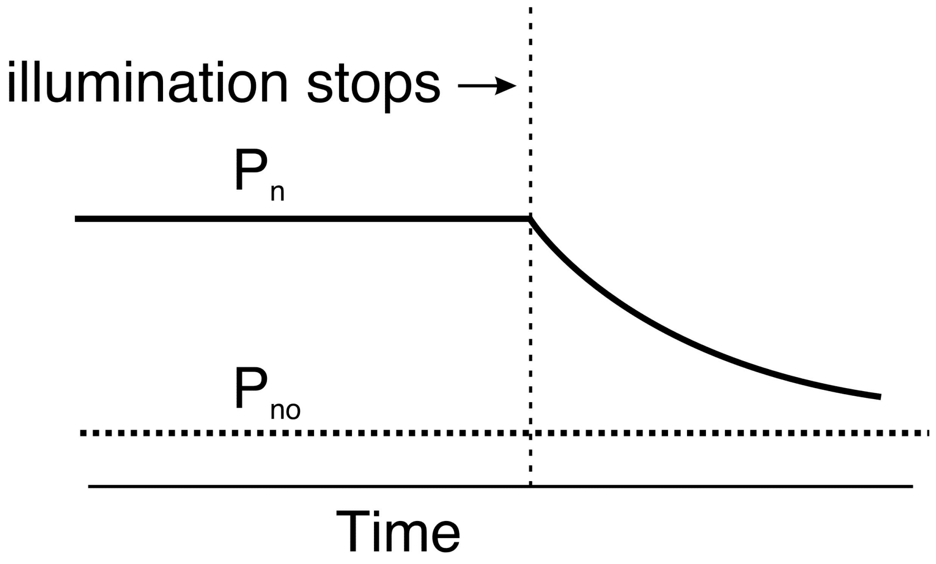
For low level injection whereΔp and p_{no} are small compared to n_{no}

$$U \approx \beta n_{no}\Delta p = \frac{p_n - p_{no}}{\frac{1}{\beta n_{no}}} \qquad U = \frac{p_n - p_{no}}{\tau_p} \text{ where } \tau_p = \beta n_n$$

Recombination rate is proportional to excess minority carrier concentration

Direct recombination III

Like a first order chemical reaction rate depends on one reactant



$$p_n(t) = p_{no} + \tau_p G_L e - \left(\frac{t}{\tau_p}\right)$$

Also because $$\tau_p = \frac{1}{\beta n_{no}}$$

Minority carrier lifetime is controlled by majority carrier concentration

Indirect recombination

For indirect band gap semiconductors like silicon

direct recombination is rare because have to lose crystal momentum as well as energy

instead recombination occurs indirectly via trapping states (defects and impurities)

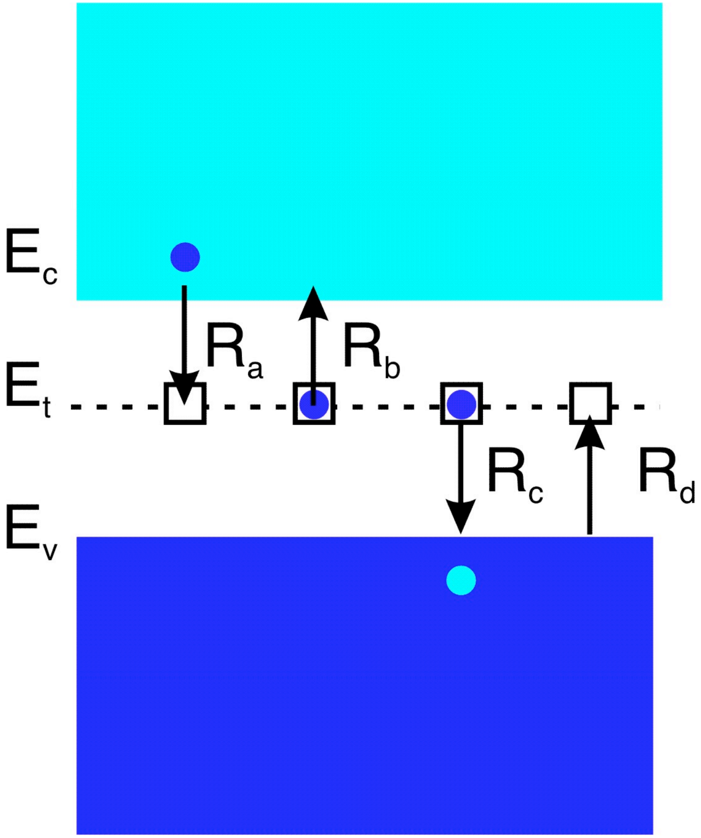
Four possible processes

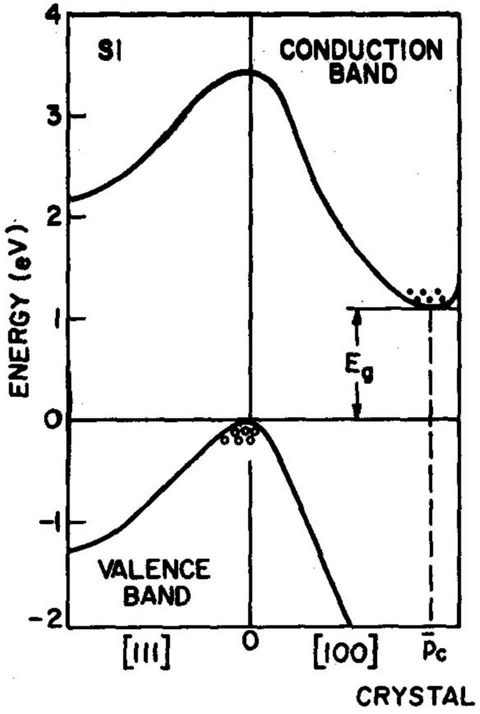
(a) Trapping of an electron

(b) Emission of a trapped electron

(c) Combination of a trapped electron and a hole

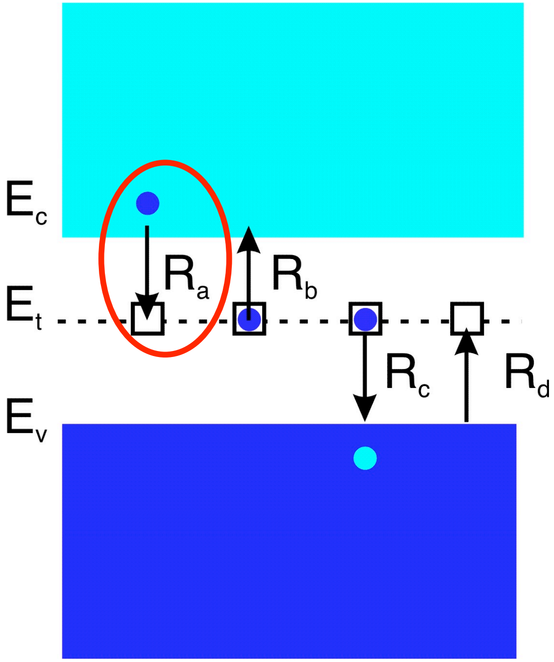
(d) Formation of a trapped electron and free hole pair





(a)

Indirect recombination II



The rate at which electrons are trapped is proportional to the number of electrons n and the number of non-occupied trapping states. The probability that a state occupied is given by the Fermi function

$$F = \frac{1}{1 + e^{\frac{(E_t - E_f)}{kT}}}$$

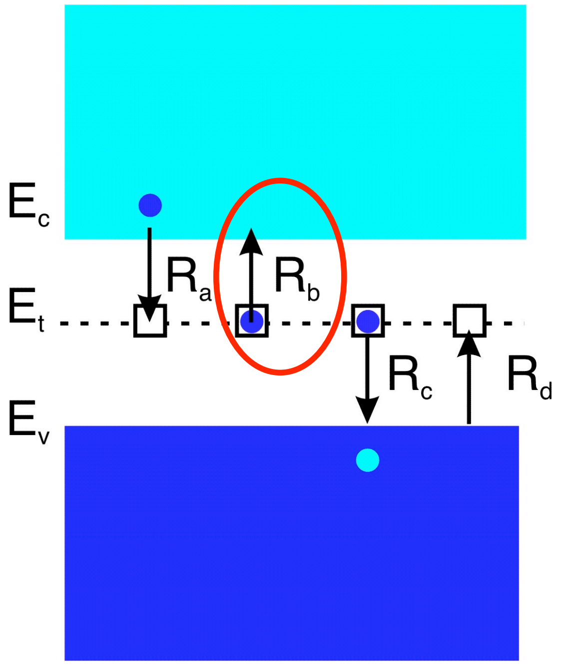
Rate will then be proportional to $nN_t(1-F)$

Or $$R_a = v_{th}\sigma_n n N_t(1-F)$$

Where the constant of proportionality is given as the electron thermal velocity times a cross section for the trapping atom (of order 10^{-15} cm^2)

Can imagine this as the volume swept by the electron in unit time. If an unfilled trapping state lies in this volume the electron is trapped

Indirect recombination III



Rate of emisson of the electron from the trapped state

$$R_b = e_n N_t F$$

For thermal equilibrium R_a=R_b so the emission probability e_n

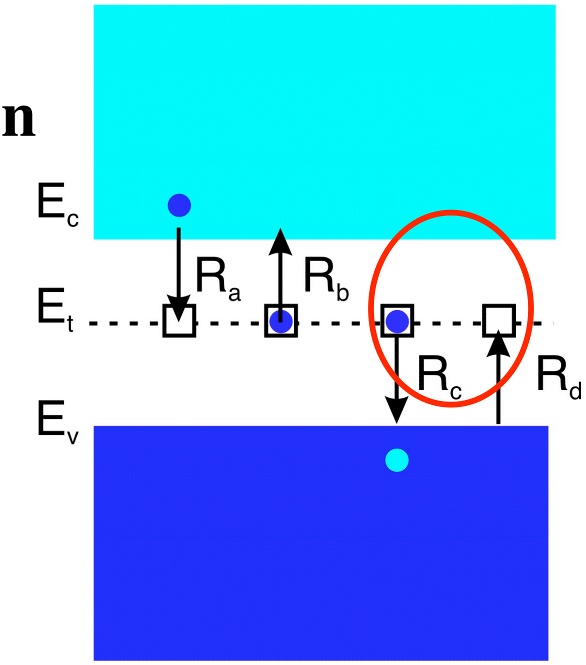
$$e_n = \frac{v_{th}\sigma_n n(1-F)}{F}$$

But $$n = n_i e^{\frac{(E_f - E_i)}{kT}}$$

And $$\frac{1-F}{F} = e^{\frac{(E_t - E_f)}{kT}}$$

so $$e_n = v_{th}\sigma_n n_i e^{\frac{(E_t - E_i)}{kT}}$$

Hole annihilation and creation



The rate of hole annihilation by a filled trapping state is analogously

$$R_c = v_{th}\sigma_p p N_t F$$

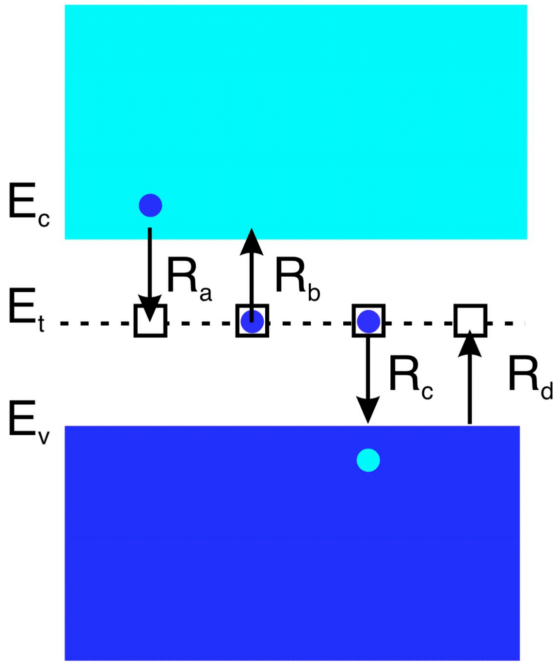
And the rate of hole emission is

$$R_d = e_p N_t (1 - F)$$

Where e_p is the emission probability which can be obtained from the thermal equilibrium condition R_c=R_d

$$e_p = v_{th}\sigma_p n_i e^{\frac{(E_i - E_t)}{kT}}$$

Net recombination rate



For steady state number of electrons leaving and entering the CB are equal

$$\frac{\partial n_n}{\partial t} = G_L - (R_a - R_b) = 0$$

Principle of detailed balance

Similarly for holes in VB

$$\frac{\partial p_n}{\partial t} = G_L - (R_c - R_d) = 0$$

Combining

$$G_L = R_a - R_b = R_c - R_d$$

Substituting for R_a etc gives

$$v_{th}\sigma_n N_t\left[n_n(1-F) - n_i e^{\frac{(E_t-E_i)}{kT}} F\right] = v_{th}\sigma_p N_t\left[p_n F - n_i e^{\frac{(E_i-E_t)}{kT}}(1-F)\right]$$

Net recombination rate

$$v_{th}\sigma_n N_t\left[n_n(1-F)-n_i e^{\frac{(E_t-E_i)}{kT}}F\right]=v_{th}\sigma_p N_t\left[p_n F-n_i e^{\frac{(E_i-E_t)}{kT}}(1-F)\right]$$

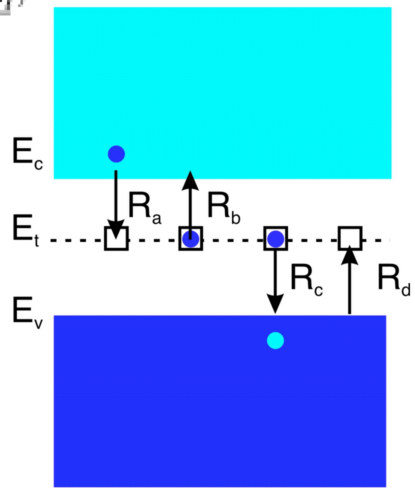
Solve this for F and then substitute back to get a net recombination rate

$$U\equiv R_a-R_b=\frac{v_{th}\sigma_n\sigma_p N_t(p_n n_n-n_i^2)}{\sigma_p\left[p_n+n_i e^{\frac{(E_i-E_t)}{kT}}\right]+\sigma_n\left[n_n+n_i e^{\frac{(E_t-E_i)}{kT}}\right]}$$

Horrible expression but can simplify since for low-level injection $n_n >> p_n$ and since the trapping states are near the centre of the gap

$$n_n \gg n_i e^{\frac{(E_t-E_i)}{kT}}$$

$$U\approx v_{th}\sigma_p N_t(p_n-p_{no})$$



Same form as for direct gap semiconductors but $\tau_p\equiv\frac{1}{v_{th}\sigma_p N_t}$

Minority carrier lifetime is now controlled by density of trapping states not majority carrier concentration

Energy dependence of recombination

$$U \equiv R_a - R_b = \frac{v_{th}\sigma_n\sigma_p N_t (p_n n_n - n_i^2)}{\sigma_p[p_n + n_i e^{\frac{(E_i - E_t)}{kT}}] + \sigma_n[n_n + n_i e^{\frac{(E_t - E_i)}{kT}}]}$$

Can also simplify this equation by assuming that the electron and hole cross sections are equal

$$U = \frac{v_{th}\sigma_0 N_t (p_n n_n - n_i^2)}{p_n + n_n + 2n_i cosh(\frac{E_t - E_i}{kT})}$$

Under low injection conditions this approximates to

$$U \approx \frac{v_{th}\sigma_0 N_t (p_n - p_{no})}{1 + (\frac{2n_i}{n_{no} + p_{no}}) cosh(\frac{E_t - E_i}{kT})}$$

Where the recombination lifetime

$$\tau_r = \frac{1 + (\frac{2n_i}{n_{no} + p_{no}}) cosh(\frac{E_t - E_i}{kT})}{v_{th}\sigma_0 N_t}$$

Carrier depletion

Can also perturb from equilibrium by removing carriers

Setting p_n and $n_n < n_i$

$$U = \frac{v_{th}\sigma_0 N_t (p_n n_n - n_i^2)}{p_n + n_n + 2n_i cosh(\frac{E_t - E_i}{kT})}$$

Becomes

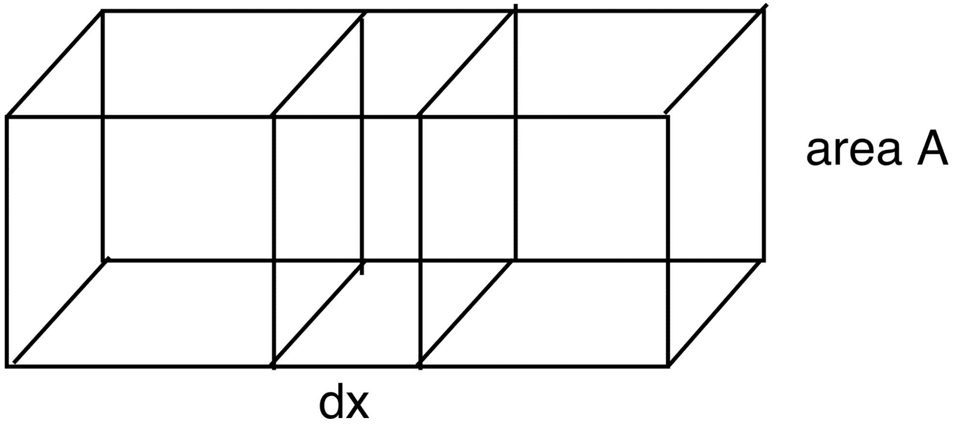
$$G = \frac{v_{th}\sigma_0 N_t n_i}{2cosh(\frac{E_t - E_i}{kT})}$$

With the generation lifetime

$$\tau_g = \frac{2cosh(\frac{E_t - E_i}{kT})}{v_{th}\sigma_0 N_t}$$

Continuity equation

We have seen the ways carriers can move under applied field and concentration gradients and that the can be created and destroyed but in general all these processes occur together



If we consider a slice of semiconductor of width dx then rate of change of the carrier density will be the sum of the currents entering from each surface and the overall generation and recombination rates.

$$\frac{\partial n}{\partial t}Adx = \left(\frac{J_n(x)A}{-q} - \frac{J_n(x+dx)A}{-q}\right) + (G_n - R_n)Adx$$

Continuity equation II

$$\frac{\partial n}{\partial t} A dx = \left(\frac{J_n(x)A}{-q} - \frac{J_n(x+dx)A}{-q}\right) + (G_n - R_n) A dx$$

Expand currents as Taylor series

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + (G_n - R_n) \text{ and } \frac{\partial p}{\partial t} = \frac{-1}{q}\frac{\partial J_p}{\partial x} + (G_p - R_p)$$

Substitute in for various forms of current and for minority carriers this becomes

$$\frac{\partial n}{\partial t} = n_p \mu_n \frac{\partial E}{\partial x} + \mu_n E \frac{\partial n_p}{\partial x} + D_n \frac{\partial^2 n_p}{\partial x^2} + G_n - \frac{n_p - n_{po}}{\tau_n}$$

$$\frac{\partial p}{\partial t} = -p_p \mu_p \frac{\partial E}{\partial x} - \mu_p E \frac{\partial p_n}{\partial x} + D_p \frac{\partial^2 p_n}{\partial x^2} + G_p - \frac{p_n - p_{no}}{\tau_p}$$

Continuity equation III

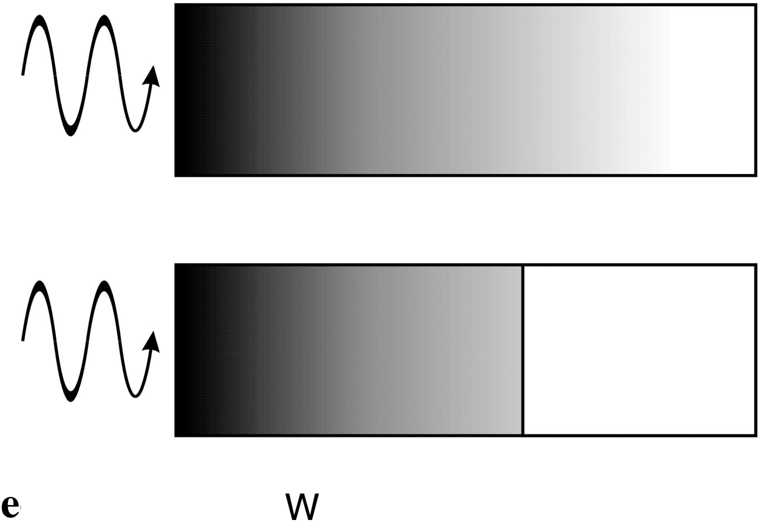
We must also satisfy Poisson's equation

$$\frac{dE}{dx} = \frac{\rho_s}{\varepsilon_s}$$

Where the charge density is the sum of the hole, electron, and ionised donor and acceptor densities taking into account their relative charges.

In general the continuity equation is difficult to solve analytically but it can be done for some special cases

Light falling on a semiconductor



Steady state with no ele

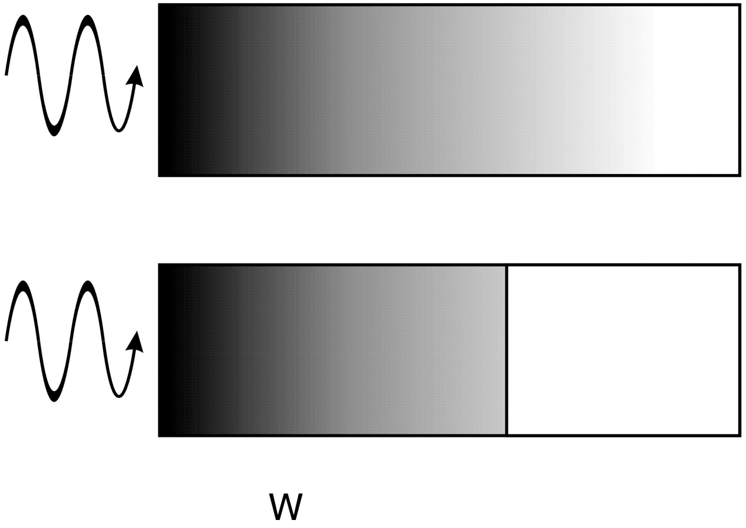
$$\frac{\partial p_n}{\partial t} = 0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{P_n - p_{no}}{\tau_p}$$

For the infinite case $p_n(0)$=constant and $p_n(\infty) = p_{no}$

$$p_n(x) = P_{no} + [p_n(0) - p_{no}] e^{\frac{-x}{L_P}}$$

diffusion length $L_p = \sqrt{D_p \tau_P}$

Extraction case



If we extract all minority carriers at a distance W the boundary condition becomes

$$p_n(W) = p_{no}$$

$$p_n(x) = P_{no} + [p_n(0) - p_{no}][\frac{sinh(\frac{W-x}{L_p})}{sinh(\frac{W}{L_p})}]$$

Diffusion current density at x=W

$$J_p = -qD_p\frac{\partial p_n}{\partial x} \text{ at x=W} = q[p_n(0) - P_{no}]\frac{D_p}{L_p}\frac{1}{sinh(\frac{W}{L_p})}$$