Semiconductor Physics

Lecture 4

Current density equations

In general we have currents flowing because of both concentration gradients and electric fields

For electrons

$$j_n = q\mu_n nE + qD_n \frac{dn}{dx}$$

For holes

$$j_p = q\mu_p nE - qD_p \frac{dp}{dx}$$

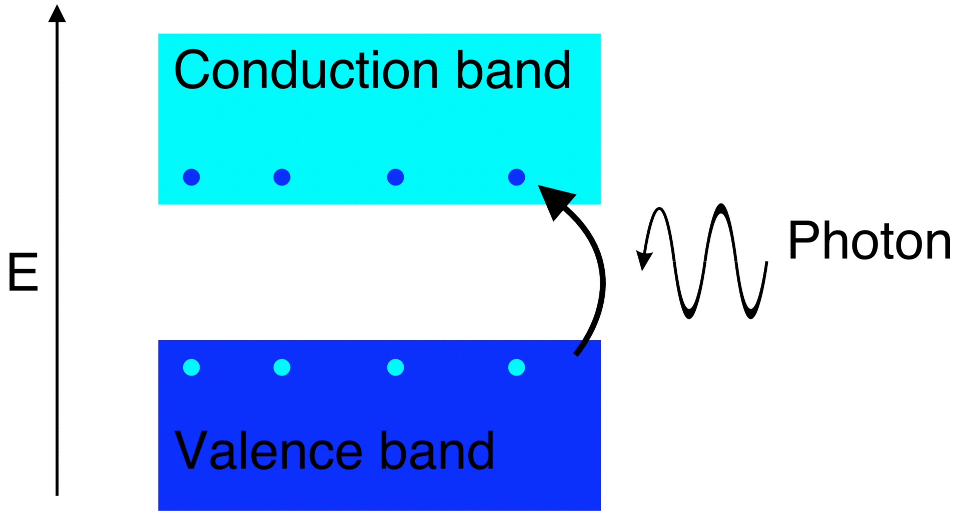
The total conductivity

$$j_{cond} = j_p + j_n$$

Carrier injection

So far we have considered cases where the semiconductor is in thermal equilibrium (at least locally) and the law of mass action holds

$$np = n_i^2 = N_c N_v e^{-(E_c - E_v)} = N_c N_v e^{\frac{-E_g}{kT}}$$



Non-equilibrium case

We can inject extra carrier by various methods

by shining light on the material

by biasing a pn junction

Magnitude of the number of carriers determines the level of injection

Carrier injection by light

Since a photon creates an electron hole pair $\Delta p = \Delta n$

Case of n-doped silicon where $N_D=10^{15}$ cm^{-3} and $n_i=10^{10}$ cm^{-3} $n=10^{15}$ cm^{-3} , $p= 10^5$ cm^{-3}

If we inject $\Delta p=\Delta n=10^{12}$ cm^{-3}

Increase p by 7 orders of magnitude

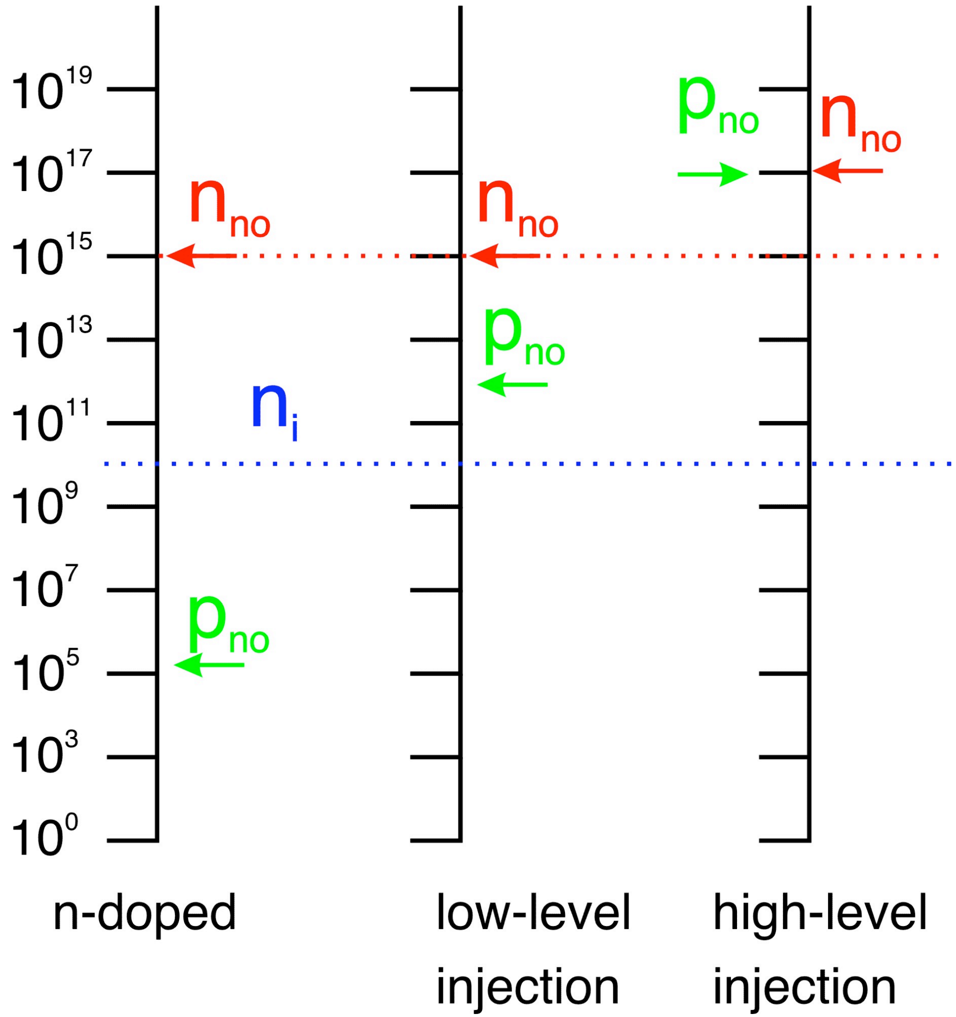
Increase n by 1%

Low level injection affects only minority carrier concentration

If $\Delta p=\Delta n=10^{17}$ cm^{-3}

Overwhelm the equilibrium majority carrier concentration

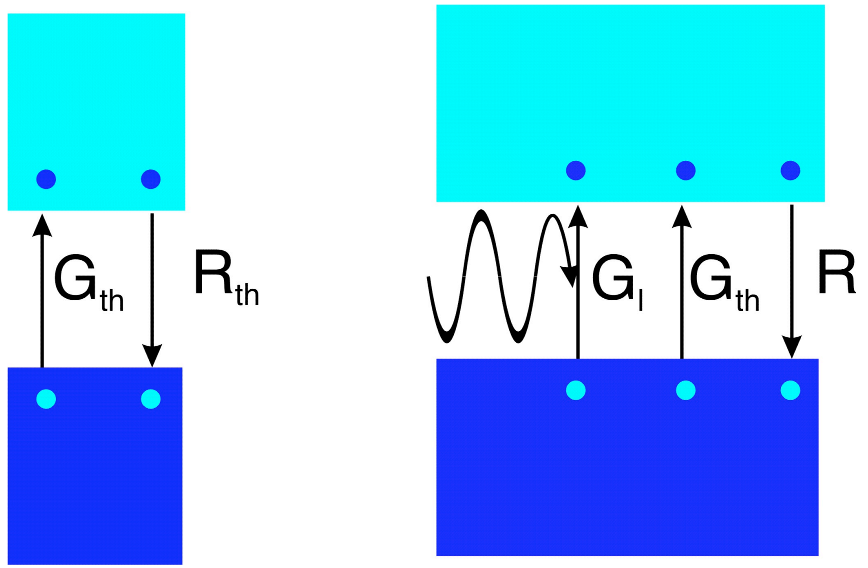
High level injection

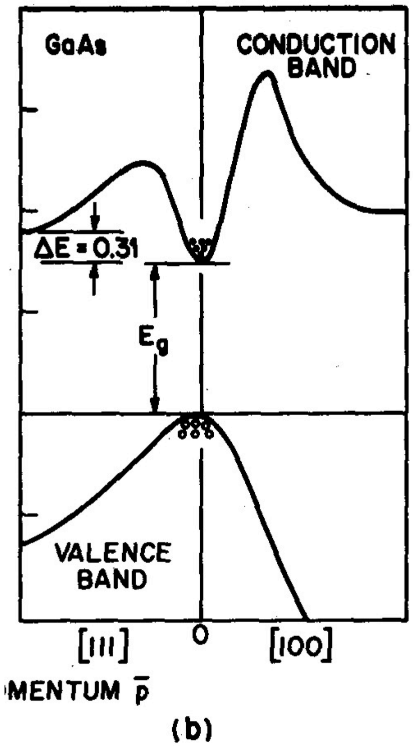


Direct recombination

Injected carriers are a non-equilibrium phenomenon and are removed by recombination of electron hole pairs

In a direct gap (like GaAs semiconductor this occurs directly electrons and holes simply combine and annihilate (sometimes with the emission of a photon)





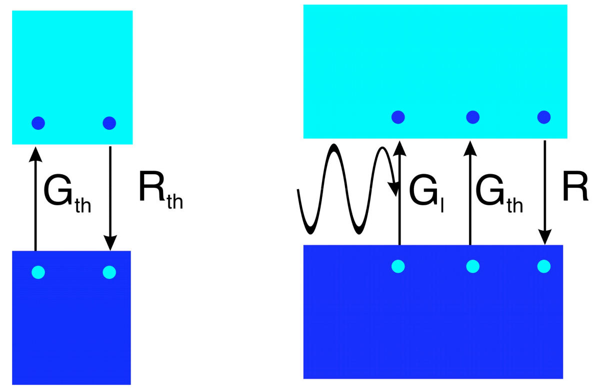
Recombination rate is proportional to the concentration of holes and electrons

$$R = \beta np$$

For thermal equilibrium case where carrier concentrations are constant

$$G_{th} = R_{th} = \beta n_{no} p_{no}$$

Direct recombination II



Rates of recombinatioon and generation are thus

$$R = \beta(n_{no} + \Delta n)(p_{no} + \Delta p)$$
$$G = G_L + G_{th}$$

Rate of change of hole concentration is given by

$$\frac{\partial p_n}{\partial t} = G - R = G_L + G_{th} - R$$

For steady state define U the net recombination rate $U = R - G_{th} = G_l$

$$G_{th} = R_{th} = \beta n_{no} p_{no} \qquad R = \beta(n_{no} + \Delta n)(p_{no} + \Delta p)$$

Recalling $\Delta n = \Delta p$

$$U = \beta(n_{no} + p_{no} + \Delta p)\Delta p$$

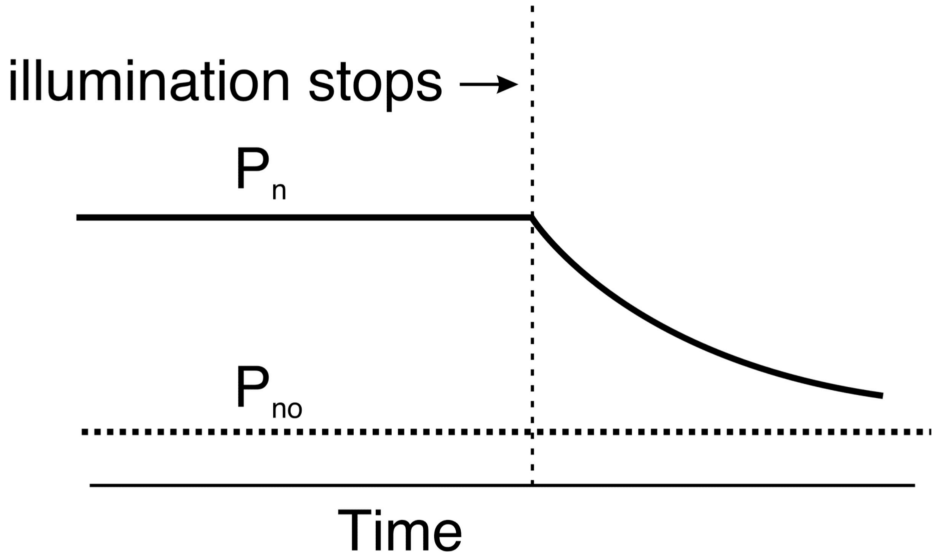
For low level injection where Δp and p_{no} are small compared to n_{no}

$$U \approx \beta n_{no} \Delta p = \frac{p_n - p_{no}}{\frac{1}{\beta n_{no}}} \qquad U = \frac{p_n - p_{no}}{\tau_p} \text{ where } \tau_p = \beta n_n$$

Recombination rate is proportional to excess minority carrier concentration

Direct recombination III

Like a first order chemical reaction rate depends on one reactant



$$p_n(t) = p_{no} + \tau_p G_L e - (\frac{t}{\tau_p})$$

Also because $$\tau_p = \frac{1}{\beta n_{no}}$$

Minority carrier lifetime is controlled by majority carrier concentration

Indirect recombination

For indirect band gap semiconductors like silicon
direct recombination is rare because have to lose crystal momentum as well as energy
instead recombination occurs indirectly via trapping states (defects and impurities)

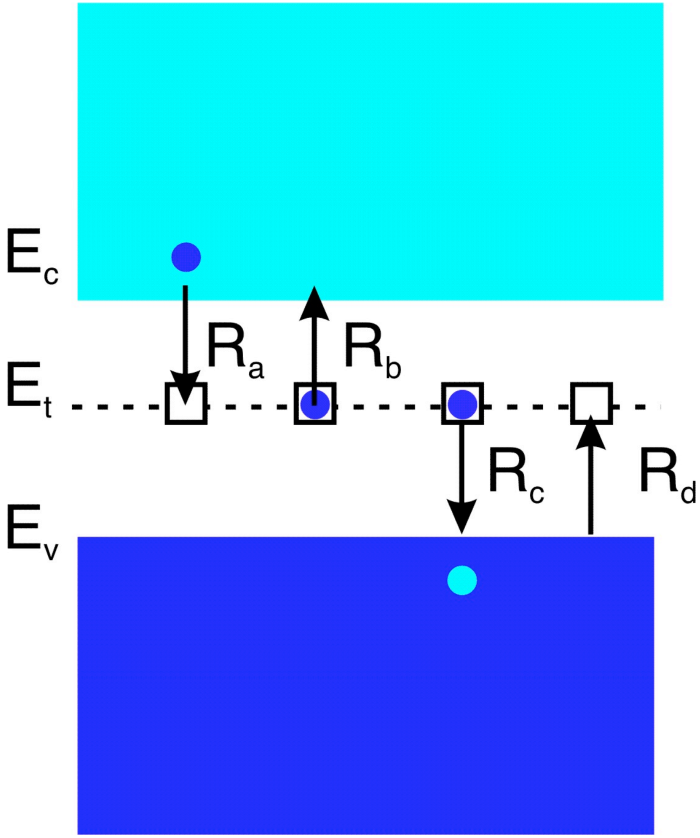
Four possible processes

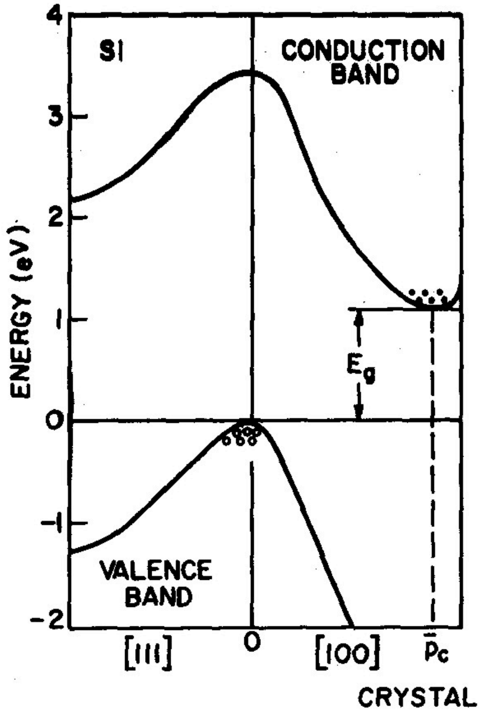
(a) Trapping of an electron

(b) Emission of a trapped electron

(c) Combination of a trapped electron and a hole

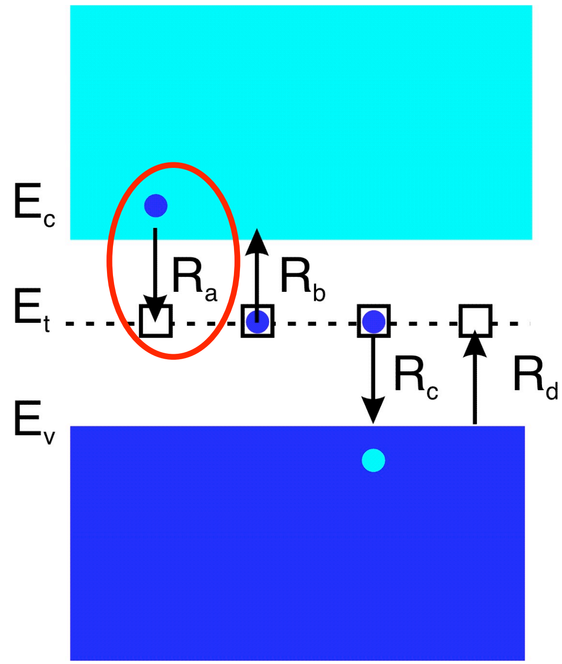
(d) Formation of a trapped electron and free hole pair





(a)

Indirect recombination II



The rate at which electrons are trapped is proportional to the number of electrons n and the number of non-occupied trapping states. The probability that a state occupied is given by the Fermi function

$$F = \frac{1}{1 + e^{\frac{(E_t - E_f)}{kT}}}$$

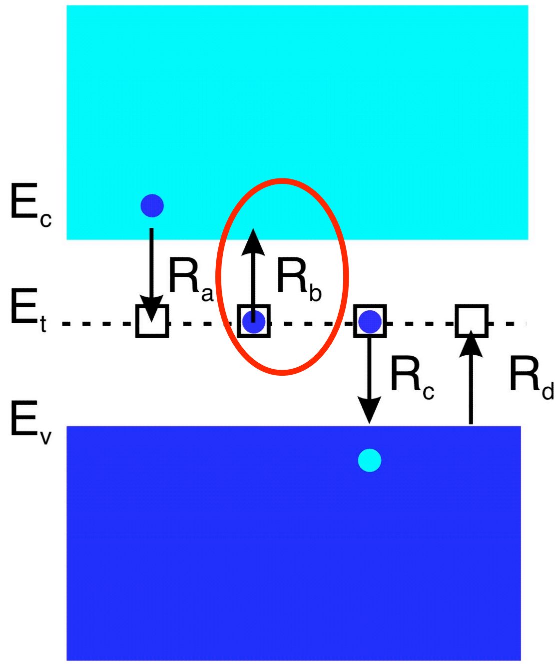
Rate will then be proportional to $nN_t(1 - F)$

Or $$R_a = v_{th}\sigma_n n N_t (1 - F)$$

Where the constant of proportionality is given as the electron thermal velocity times a cross section for the trapping atom (of order 10^{-15} cm^2)

Can imagine this as the volume swept by the electron in unit time. If an unfilled trapping state lies in this volume the electron is trapped

Indirect recombination III



Rate of emisson of the electron from the trapped state

$$R_b = e_n N_t F$$

For thermal equilibrium R_a=R_b so the emission probability e_n

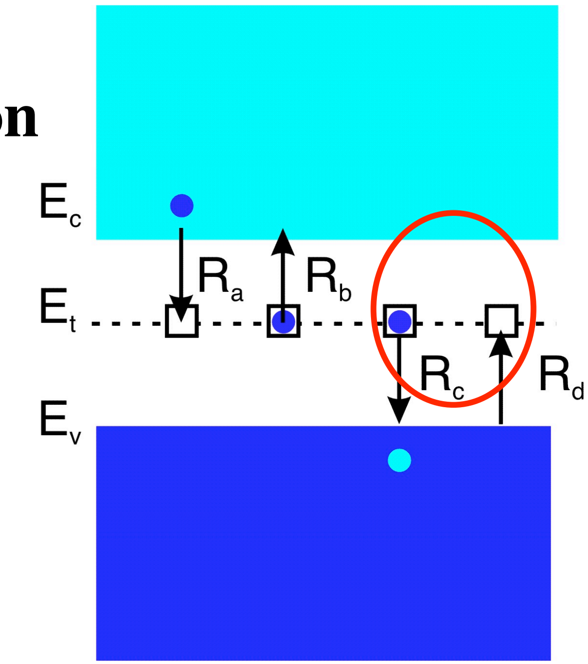
$$e_n = \frac{v_{th}\sigma_n n(1-F)}{F}$$

But $$n = n_i e^{\frac{(E_f - E_i)}{kT}}$$

And $$\frac{1-F}{F} = e^{\frac{(E_t - E_f)}{kT}}$$

so $$e_n = v_{th}\sigma_n n_i e^{\frac{(E_t - E_i)}{kT}}$$

Hole annihilation and creation



The rate of hole annihilation by a filled trapping state is analogously

$$R_c = v_{th}\sigma_p p N_t F$$

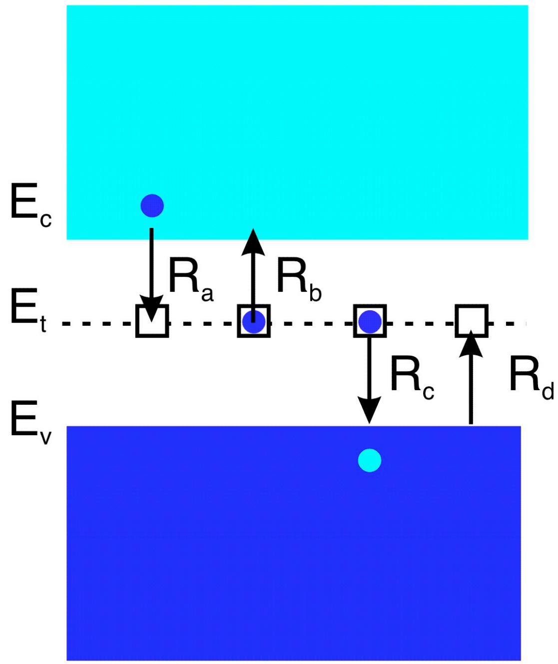
And the rate of hole emission is

$$R_d = e_p N_t (1 - F)$$

Where e_p is the emission probability which can be obtained from the thermal equilibrium condition $R_c = R_d$

$$e_p = v_{th}\sigma_p n_i e^{\frac{(E_i - E_t)}{kT}}$$

Net recombination rate



For steady state number of electrons leaving and entering the CB are equal

$$\frac{\partial n_n}{\partial t} = G_L - (R_a - R_b) = 0$$

Principle of detailed balance

Similarly for holes in VB

$$\frac{\partial p_n}{\partial t} = G_L - (R_c - R_d) = 0$$

Combining

$$G_L = R_a - R_b = R_c - R_d$$

Substituting for R_a etc gives

$$v_{th}\sigma_n N_t[n_n(1-F) - n_i e^{\frac{(E_t - E_i)}{kT}} F] = v_{th}\sigma_p N_t[p_n F - n_i e^{\frac{(E_i - E_t)}{kT}}(1-F)]$$

Net recombination rate

$$v_{th}\sigma_n N_t[n_n(1-F) - n_i e^{\frac{(E_t-E_i)}{kT}} F] = v_{th}\sigma_p N_t[p_n F - n_i e^{\frac{(E_i-E_t)}{kT}}(1-F)]$$

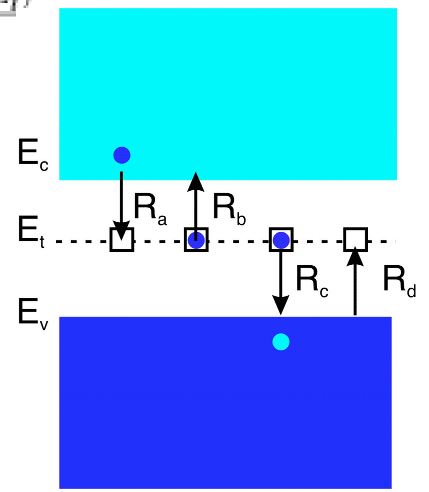
Solve this for F and then substitute back to get a net recombination rate

$$U \equiv R_a - R_b = \frac{v_{th}\sigma_n\sigma_p N_t(p_n n_n - n_i^2)}{\sigma_p[p_n + n_i e^{\frac{(E_i-E_t)}{kT}}] + \sigma_n[n_n + n_i e^{\frac{(E_t-E_i)}{kT}}]}$$

Horrible expression but can simplify since for low-level injection $n_n >> p_n$ and since the trapping states are near the centre of the gap

$$n_n \gg n_i e^{\frac{(E_t-E_i)}{kT}}$$

$$U \approx v_{th}\sigma_p N_t(p_n - p_{no})$$



Same form as for direct gap semiconductors but $\tau_p \equiv \frac{1}{v_{th}\sigma_p N_t}$

Minority carrier lifetime is now controlled by density of trapping states not majority carrier concentration

Energy dependence of recombination

$$U \equiv R_a - R_b = \frac{v_{th}\sigma_n\sigma_p N_t(p_n n_n - n_i^2)}{\sigma_p[p_n + n_i e^{\frac{(E_i-E_t)}{kT}}] + \sigma_n[n_n + n_i e^{\frac{(E_t-E_i)}{kT}}]}$$

Can also simplify this equation by assuming that the electron and hole cross sections are equal

$$U = \frac{v_{th}\sigma_0 N_t(p_n n_n - n_i^2)}{p_n + n_n + 2n_i cosh(\frac{E_t-E_i}{kT})}$$

Under low injection conditions this approximates to

$$U \approx \frac{v_{th}\sigma_0 N_t(p_n - p_{no})}{1 + (\frac{2n_i}{n_{no}+p_{no}})cosh(\frac{E_t-E_i}{kT})}$$

Where the recombination lifetime

$$\tau_r = \frac{1 + (\frac{2n_i}{n_{no}+p_{no}})cosh(\frac{E_t-E_i}{kT})}{v_{th}\sigma_0 N_t}$$

Carrier depletion

Can also perturb from equilibrium by removing carriers
Setting p_n and $n_n < n_i$

$$U = \frac{v_{th}\sigma_0 N_t(p_n n_n - n_i^2)}{p_n + n_n + 2n_i cosh(\frac{E_t - E_i}{kT})}$$

Becomes

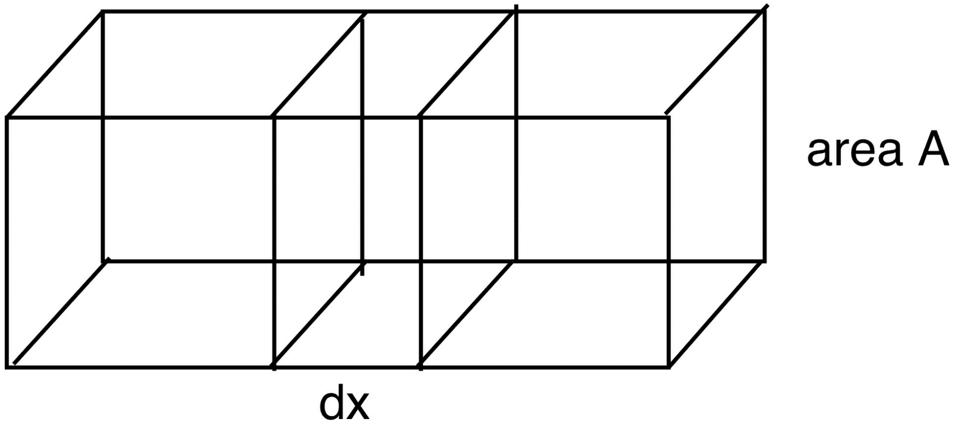
$$G = \frac{v_{th}\sigma_0 N_t n_i}{2cosh(\frac{E_t - E_i}{kT})}$$

With the generation lifetime

$$\tau_g = \frac{2cosh(\frac{E_t - E_i}{kT})}{v_{th}\sigma_0 N_t}$$

Continuity equation

We have seen the ways carriers can move under applied field and concentration gradients and that the can be created and destroyed but in general all these processes occur together



If we consider a slice of semiconductor of width dx then rate of change of the carrier density will be the sum of the currents entering from each surface and the overall generation and recombination rates.

$$\frac{\partial n}{\partial t} A dx = \left(\frac{J_n(x)A}{-q} - \frac{J_n(x+dx)A}{-q}\right) + (G_n - R_n) A dx$$

Continuity equation II

$$\frac{\partial n}{\partial t}Adx = (\frac{J_n(x)A}{-q} - \frac{J_n(x+dx)A}{-q}) + (G_n - R_n)Adx$$

Expand currents as Taylor series

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + (G_n - R_n) \text{ and } \frac{\partial p}{\partial t} = \frac{-1}{q}\frac{\partial J_p}{\partial x} + (G_p - R_p)$$

Substitute in for various forms of current and for minority carriers this becomes

$$\frac{\partial n}{\partial t} = n_p\mu_n\frac{\partial E}{\partial x} + \mu_n E\frac{\partial n_p}{\partial x} + D_n\frac{\partial^2 n_p}{\partial x^2} + G_n - \frac{n_p - n_{po}}{\tau_n}$$

$$\frac{\partial p}{\partial t} = -p_p\mu_p\frac{\partial E}{\partial x} - \mu_p E\frac{\partial p_n}{\partial x} + D_p\frac{\partial^2 p_n}{\partial x^2} + G_p - \frac{p_n - p_{no}}{\tau_p}$$

Continuity equation III

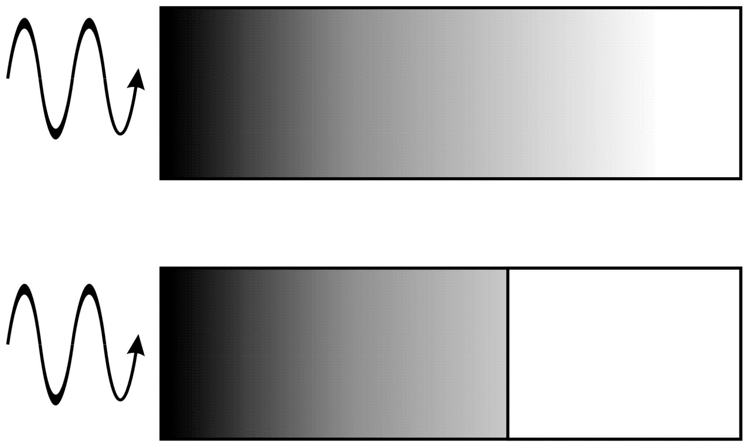
We must also satisfy Poisson's equation

$$\frac{dE}{dx} = \frac{\rho_s}{\varepsilon_s}$$

Where the charge density is the sum of the hole, electron, and ionised donor and acceptor densities taking into account their relative charges.

In general the continuity equation is difficult to solve analytically but it can be done for some special cases

Light falling on a semiconductor



Steady state with no ele

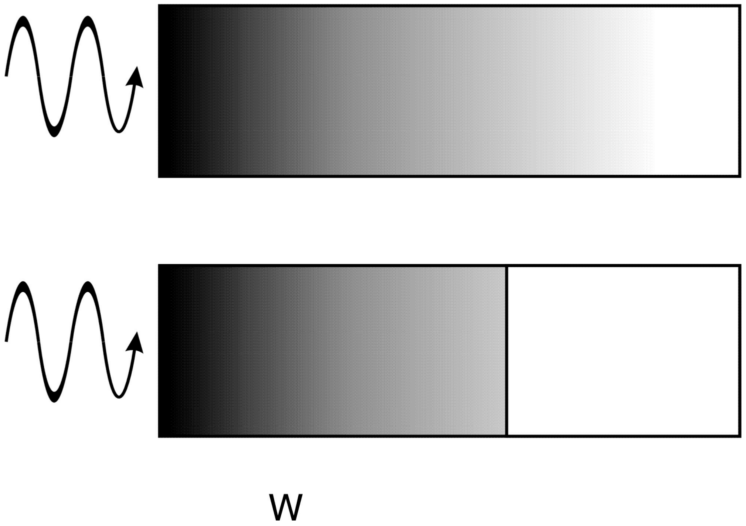
$$\frac{\partial p_n}{\partial t} = 0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{P_n - p_{no}}{\tau_p}$$

For the infinite case $p_n(0)$=constant and $p_n(\infty)= p_{no}$

$$p_n(x) = P_{no} + [p_n(0) - p_{no}]e^{\frac{-x}{L_P}}$$

diffusion length $L_p = \sqrt{D_p \tau_P}$

Extraction case



If we extract all minority carriers at a distance W the boundary condition becomes

$$p_n(W) = p_{no}$$

$$p_n(x) = P_{no} + [p_n(0) - p_{no}]\left[\frac{sinh(\frac{W-x}{L_p})}{sinh(\frac{W}{L_p})}\right]$$

Diffusion current density at x=W

$$J_p = -qD_p \frac{\partial p_n}{\partial x} \text{ at x=W} = q[p_n(0) - P_{no}]\frac{D_p}{L_p}\frac{1}{sinh(\frac{W}{L_p})}$$