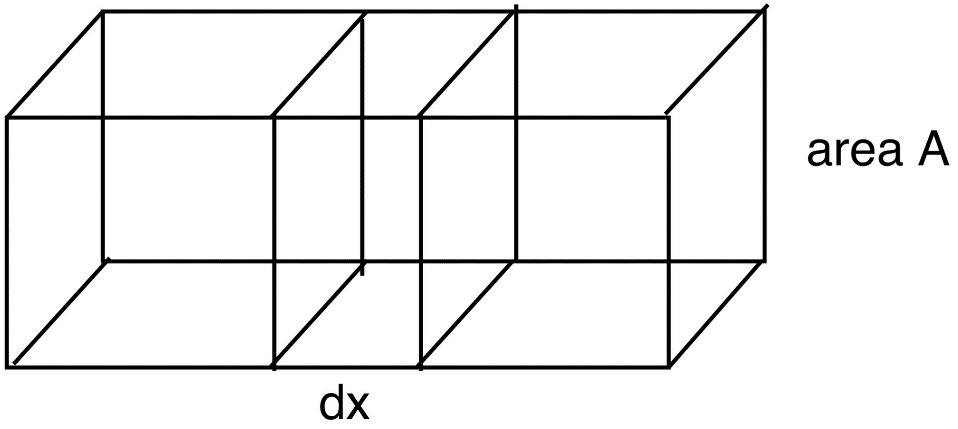
Semiconductor Physics

Lecture 5

Recap continuity equation

We have seen the ways carriers can move under applied field and concentration gradients and that the can be created and destroyed but in general all these processes occur together



If we consider a slice of semiconductor of width dx then rate of change of the carrier density will be the sum of the currents entering from each surface and the overall generation and recombination rates.

$$\frac{\partial n}{\partial t}Adx = \left(\frac{J_n(x)A}{-q} - \frac{J_n(x+dx)A}{-q}\right) + (G_n - R_n)Adx$$

Continuity equation II

$$\frac{\partial n}{\partial t}Adx = (\frac{J_n(x)A}{-q} - \frac{J_n(x+dx)A}{-q}) + (G_n - R_n)Adx$$

Expand currents as Taylor series

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + (G_n - R_n) \text{ and } \frac{\partial p}{\partial t} = \frac{-1}{q}\frac{\partial J_p}{\partial x} + (G_p - R_p)$$

$$J_n = n_p \mu_n E$$

Substitute in for various forms of current and for minority carriers this becomes

$$\frac{\partial n}{\partial t} = n_p \mu_n \frac{\partial E}{\partial x} + \mu_n E \frac{\partial n_p}{\partial x} + D_n \frac{\partial^2 n_p}{\partial x^2} + G_n - \frac{n_p - n_{po}}{\tau_n}$$

$$\frac{\partial p}{\partial t} = -p_p \mu_p \frac{\partial E}{\partial x} - \mu_p E \frac{\partial p_n}{\partial x} + D_p \frac{\partial^2 p_n}{\partial x^2} + G_p - \frac{p_n - p_{no}}{\tau_p}$$

Continuity equation III

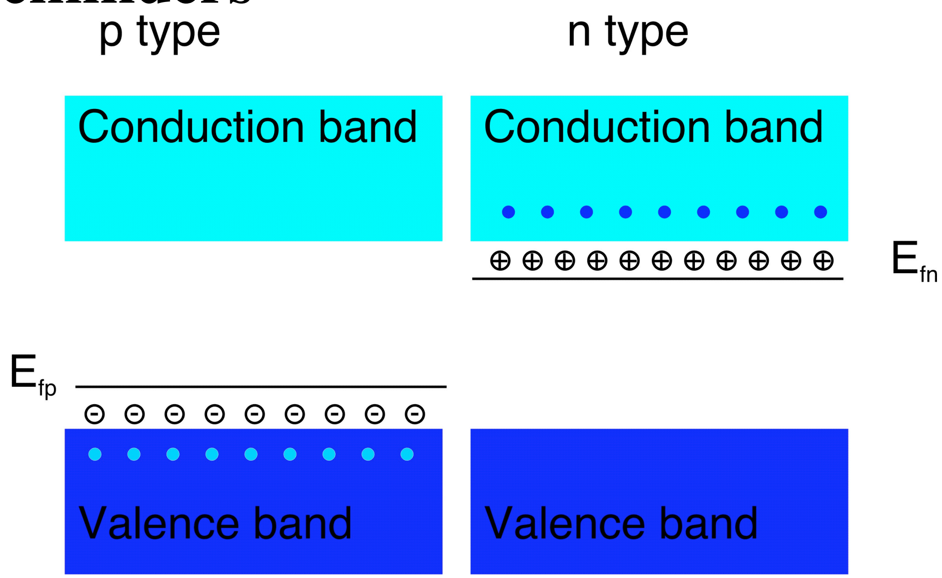
We must also satisfy Poisson's equation

$$\frac{dE}{dx} = \frac{\rho_s}{\varepsilon_s}$$

Where the charge density is the sum of the hole, electron, and ionised donor and acceptor densities taking into account their relative charges.

In general the continuity equation is difficult to solve analytically but it can be done for some special cases

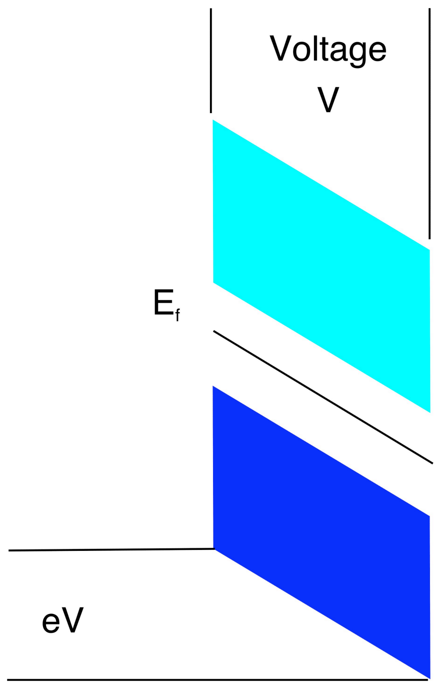
Reminders



In lecture three we met the idea that the Fermi level is strongly affected by the doping level

New concept

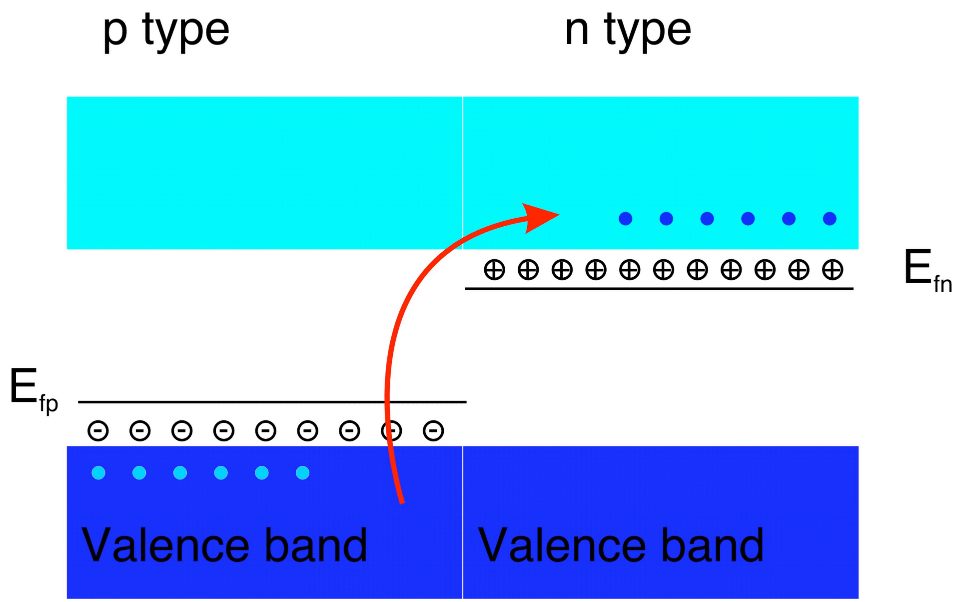
applying a voltage across a semiconductor means that all energy levels have a gradient



P N junction

When a joint is formed between p and n type semi conductor the Fermi energy must become constant through the boundary. Have a motion of carriers across the boundary.

Leads to uncompensated donor and acceptor ions in a region close to the interface known as the depletion region



Depletion region

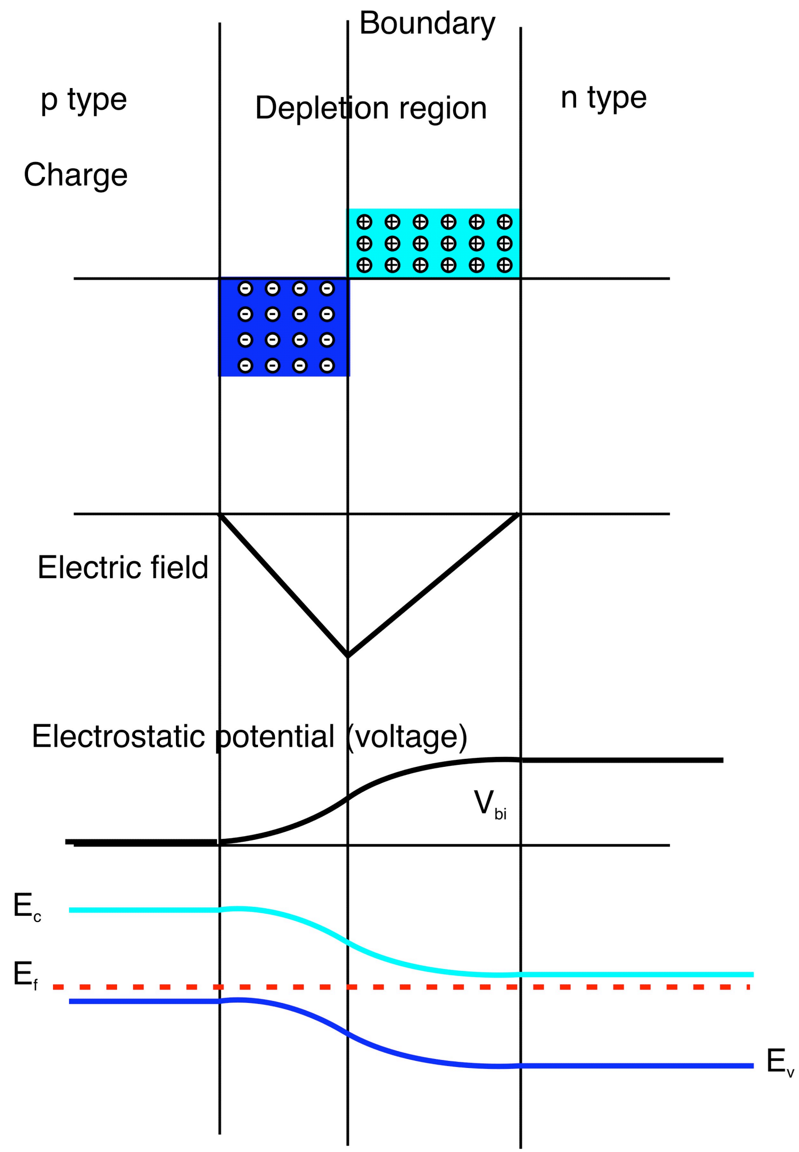
Depletion region is so called because of the lack of free carriers

To keep overall charge neutrality have equal numbers of uncompensated donors and acceptors

Charge separation leads to an electric field in the depletion region

And a voltage V_{bi} across the pn junction

Voltage bends the bands so that the Fermi energy remains constant across the boundary moving from being just above E_v to just below E_c



Fermi- energy and the pn junction

The Fermi energy is a measure of the ability of the electrons to do work it is the chemical potential

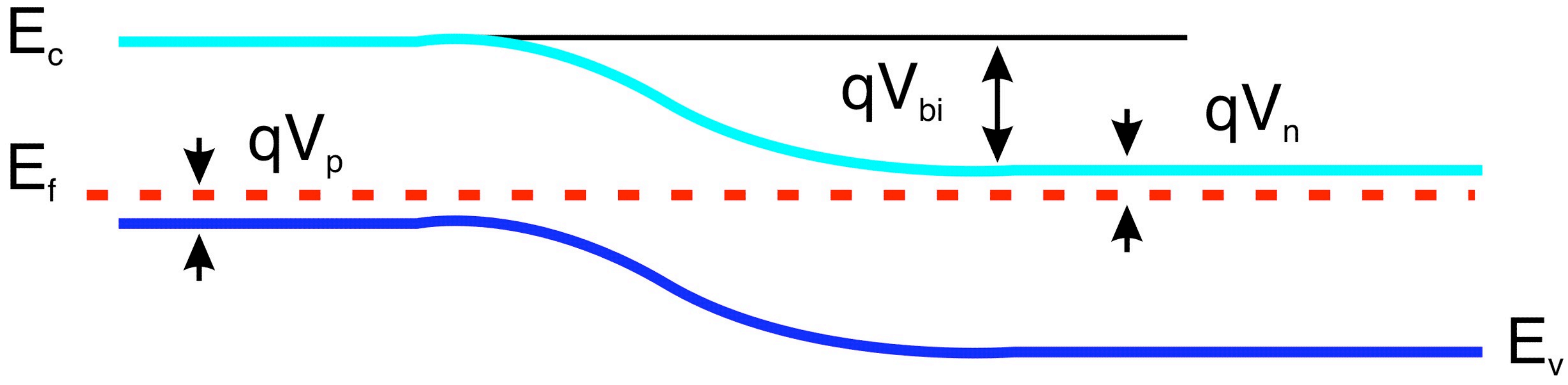
$$F(E) = \frac{1}{1 + e^{\frac{(E-E_f)}{kT}}}$$

For a pn junction in thermal equilibrium no currents are flowing so the Fermi energy is constant through the material

$$\frac{\partial E_f}{\partial x} = J_n = 0 = \mu_n (n\xi + \frac{kT}{q}\frac{\partial n}{\partial x})$$

Diffusion current is balanced by current cause by electric field (for both electrons and holes)

Potentials



Built-in potential is qV_b

$$qV_{bi} = E_g - q(V_n + V_p) = E_g - (E_{fp} - E_v) - (E_c - E_f)$$

Where E_{fp} and E_{fn} are the fermi energies in isolated p and n type material
Recall that for n-type semiconductor while for p type

$$E_c - E_f = kT\,ln(\frac{N_c}{N_D}) \qquad E_f - E_v = kT\,ln(\frac{N_v}{N_a})$$

$$E_g = kT\,ln(\frac{N_c N_v}{n_i^2})$$

Potentials II

$$V_{bi} = \frac{kT}{q} ln(\frac{N_a N_d}{n_i^2})$$

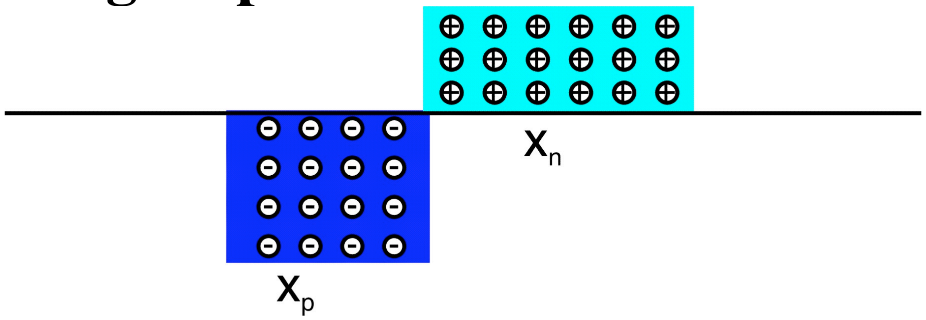
Since at equilibrium $n_{po}p_{po} = n_{no}p_{no} = n_i^2$

$$V_{bi} = \frac{kT}{q} ln(\frac{p_{po}}{p_{no}}) = \frac{kT}{q} ln(\frac{n_{no}}{n_{po}})$$

And

$$p_{no} = p_{po}e - (\frac{qV_{bi}}{kT}) \text{ and } n_{po} = p_{no}e - (\frac{qV_{bi}}{kT})$$

Charge separation and electric field



For abrupt boundaries since we have charge neutrality

$$N_a x_p = N_d x_n$$

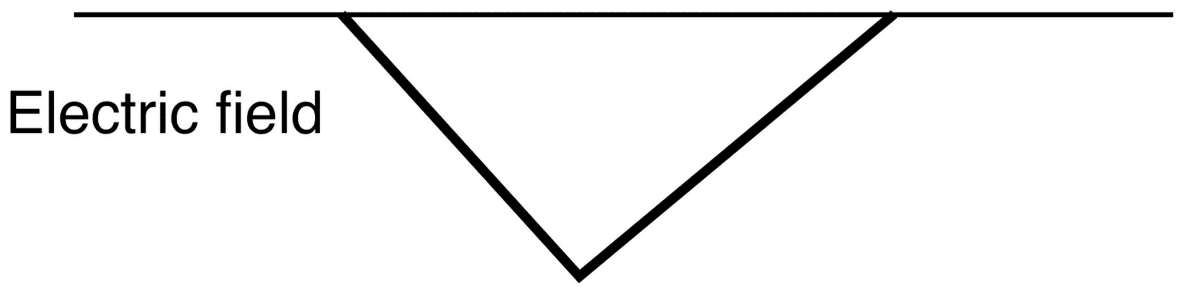
Poisson's equation states

$$\frac{\partial \xi}{\partial x} = \frac{\rho(x)}{\varepsilon_s} = \frac{q}{\varepsilon_s}(p(x) - n(x) + N_d(x) - N_a(x))$$

$$\frac{\partial \xi}{\partial x} \approx \frac{q}{\varepsilon_s} N_d \text{ for } 0 < x \leq x_n$$

$$\frac{\partial \xi}{\partial x} \approx \frac{-q}{\varepsilon_s} N_a \text{ for } -x_p \leq x < 0$$

Electric field



Linear electric field

$$\xi = \frac{q}{\varepsilon_s} N_a (x + x_p) \text{ for } -x_p \leq x < 0$$

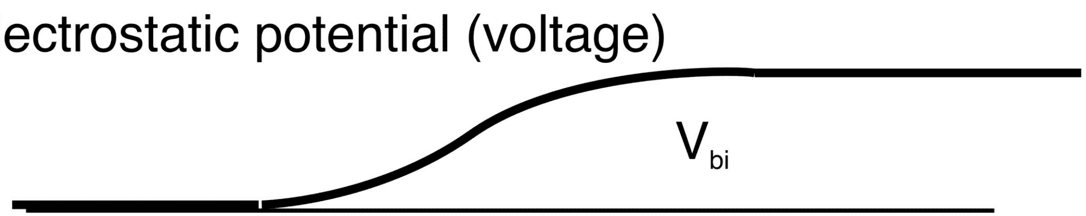
$$\xi = \frac{q}{\varepsilon_s} N_d x - \xi_m \text{ for } 0 < x \leq x_n$$

$$\xi_m = \frac{q N_d x_n}{\varepsilon_s} = \frac{q N_a x_p}{\varepsilon_s}$$

Electric potential

Still don't know width (W) of depletion layer

Integrate electric field wrt x to get potential



$$V(x) = \xi_m (x - \frac{x^2}{2W})$$

$$V_{bi} = \frac{1}{2}\xi_m W$$

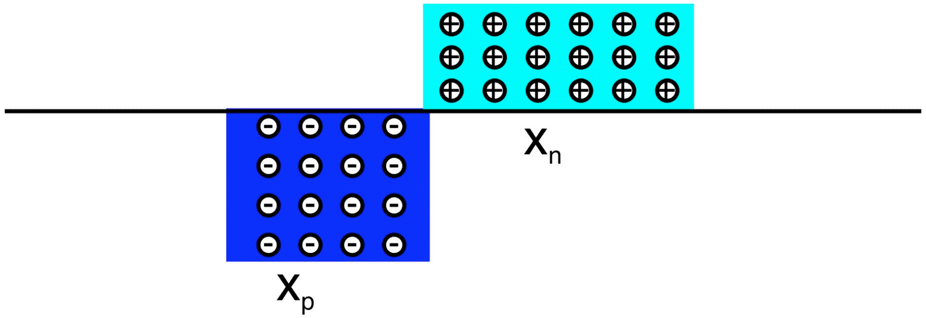
$$\xi_m = \frac{qN_d x_n}{\varepsilon_s} = \frac{qN_a x_p}{\varepsilon_s}$$

$$W = \sqrt{\frac{2\varepsilon_s}{q}(\frac{N_a + N_d}{N_a N_d})V_{bi}}$$

Depletion layer

In general junctions are formed from heavily doped n (or) and lightly doped p (or n)

$$W = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_b}}$$



Where N_b is the dominant dopant

This is a simplistic view which ignores the majority carrier contribution which tend to diffuse into the depletion region

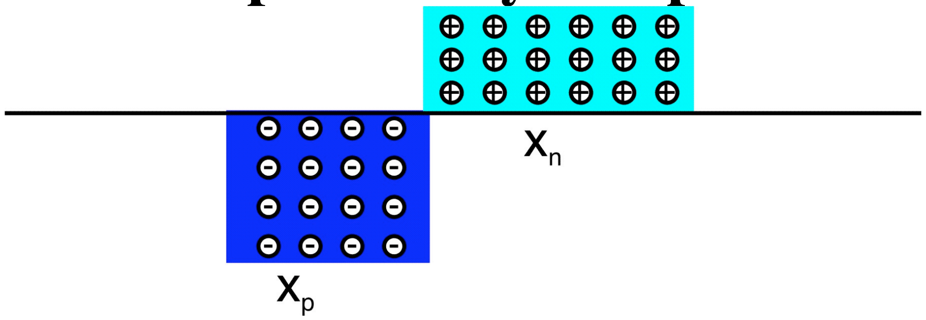
$$W = \sqrt{\frac{2\varepsilon_s (V_{bi} - \frac{2kT}{q})}{qN_b}} = L_D \sqrt{2(\beta V_{bi} - 2)}$$

Where

$$\text{Debye length } L_d = \sqrt{\frac{\varepsilon_s kT}{q^2 N_b}} \text{ and } \beta = \frac{q}{kT}$$

W is typically 5 to 10 times L_d

Depletion layer capacitance



We have a charge separation and hence there is a capacitance associated with a pn junction

$$C \equiv \frac{dQ}{dV} = \frac{d(qN_bW)}{d[(qN_b/2\varepsilon_s)W^2]} = \sqrt{\frac{q\varepsilon_s N_b}{2}}(V_{bi} \pm V - \frac{2kT}{q})^{\frac{-1}{2}}$$

$$C = \frac{\varepsilon_s}{\sqrt{2}L_d}({}_{bi} \pm \beta V - 2)^{\frac{-1}{2}}$$