Reading Assignment: Read the relevant sections of Electromagnetic Field Theory by Bo Thide

Problem 1

The gradient, curl, and divergence operations have simple relationships that will be used throughout the subject.

a. One might be tempted to apply the divergence theorem to the surface integral in Stokes' theorem. However, the divergence theorem requires a closed surface while Stokes' theorem is true in general for an open surface. Stokes' theorem for a closed surface requires the contour to shrink to zero giving a zero result for the line integral. Use the divergence theorem applied to the closed surface with vector $\nabla \times \vec{A}$ to prove that $\nabla \bullet (\nabla \times \vec{A}) = 0$.
b. Verify (a) by direct computation in cylindrical coordinates.
c. Integrate the normal component of the vector $\nabla \times (\nabla f)$ over a surface and use Stokes' theorem to show that

$$\int_S \nabla \times (\nabla f) \bullet d\overline{S} = \oint_C \nabla f \bullet d\overline{\ell} = \oint_C df = 0$$

where f(x, y, z) is an arbitrary scalar function.

Hint: $df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz = \nabla f \bullet d\overline{\ell}$

Since the equality is true for any surface $d\overline{S}$ conclude that $\nabla \times (\nabla f) = 0$.

d. Verify the results of part (c) that $\nabla \times (\nabla f) = 0$ by direct computation in spherical coordinates.

Problem 2

A cylinder of radius R_1 has a volume current distribution

$$J_z(r) = J_0(r/R_1)^2 \qquad 0 < r < R_1$$

The cylinder is surrounded by free space and a perfect conducting cylinder of radius R_2 so that $\bar{H} = 0$ for $R > R_2$. There is no surface current on the $r = R_1$ surface.

a. What is the total z directed current flowing through the cylinder?
b. What is the magnetic field $\bar{H}$ for $0 < r < R_2$?
c. What is the surface current density on the cylinder of radius R_2?
d. What is the total z directed current flowing on the $r = R_2$ cylinder and how is it related to your answer in (a)?

Problem 3

A sphere of radius R_1 and free space permittivity ε_0 has a volume charge distribution

$$\rho_f(r) = \rho_0(r/R_1)^4 \qquad 0 < r < R_1$$

The sphere is surrounded by free space and a perfectly conducting sphere of radius R_2 so that $\bar{E} = 0$ for $r > R_2$. There is no surface charge on the $r = R_1$ surface.

a. What is the total charge on the sphere?
b. What is the electric field $\bar{E}$ for $0 < r < R_2$?
c. What is the surface charge density on the perfectly conducting sphere of radius R_2?
d. What is the total charge on the $r = R_2$ spherical surface and how is it related to your answer in (a)?

Problem 4

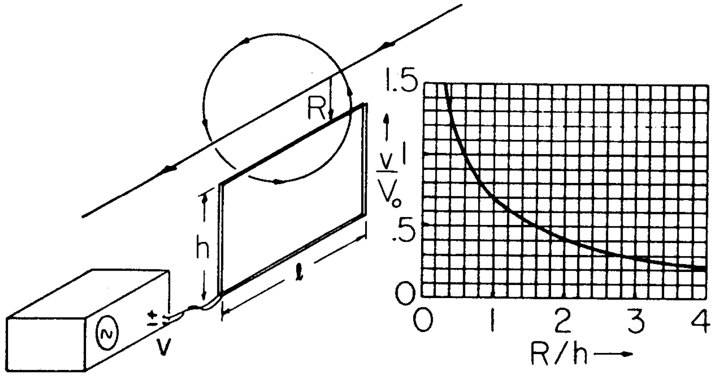

Figure 1.6.4 Demonstration of voltmeter reading induced at terminals of a coil in accordance with Faraday's law. To plot data on graph, normalize voltage to V_o as defined with (11). Because I is the peak current, v is the peak voltage.

Figure 1.6.4 in *Electromagnetic Fields and Energy*, by Hermann A. Haus and James R. Melcher, 1989.

The N turn rectangular coil of height h and length l shown above is used to measure the magnetic field intensity $\bar{H}$ due to the current $i = I_0 \sin \omega t$ in the infinitely long wire of height R above the coil. The N turn coil is open circuited and thus its current is zero.

a. What is the total magnetic flux

$$\lambda_f = \mu_0 N \int_z^{z+\ell} dz \int_R^{R+h} H_\phi dr$$

linked by the N turn coil?

b. With wire current $i = I_0 \sin \omega t$ what is $v(t)$ across the terminals of the N turn coil?

   Evaluate for N=20 turns, h=8 cm, l=20cm, I_0 = 6 amp peak, and $\omega = 120\pi$ radians (50 Hertz).

c. How should the N turn coil be positioned with respect to the line current i so that $v(t) = 0$?