Student Matriculation No:

Name:

## EEG 814: Electromagnetic Theory Assignment

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## Problem 1

a. For a particular electromagnetic field, the Cartesian component of the electric field vector are given as:

$$E_x = E_y = 0$$
  $E_z = E_0 \cos(\alpha x) \cos(\omega t)$ 

Given that the magnetic field strength at time t = 0 is  $\overline{H} = 0$ , show that

$$H_x = H_z = 0$$
  $H_y = H_0 \sin(\alpha x) \sin(\omega t)$ 

- 1. Determine  $H_0$  in terms of  $E_0$  and the permeability  $\mu$  of the medium in which the field exists.
- b. The Cartesian (x, y, z) components of the electric vector of a particular wave field propagating in an ideal dielectric medium of permeability  $\mu_0$  and permittivity  $\varepsilon$  are

$$E_x = 0;$$
  $E_y = E_0 \sin(\omega t - \alpha x);$   $E_z = E_0 \cos(\omega t - \alpha x)$ 

Where  $E_0$ ,  $\omega$ ,  $\alpha$  are constants.

- 1. What is the state of polarization of this wave field?
- 2. Obtain expressions for the Cartesian component of the magnetic field strength,  $\overline{H}$ .
- 3. Show that the Poynting vector for the wave is independent of time and the spatial co-ordinates

## Problem 2

- a. Two submerged submarines are using a 10 kHz plane electromagnetic wave for their communication. The magnitude of the electric field at the transmitter is 100mV/m, whereas the receiver requires at least 1mV/m (peak value) for reliable communication. Assuming that the conductivity and the dielectric constant of the seawater are 4 S/m and 81, respectively, find
  - 1. The wavelength
  - 2. The attenuation constant
  - 3. The phase velocity
  - 4. The skin depth of the wave, and
  - 5. The maximum range over which a reliable communication is possible

b. The region x > 0 is a perfect dielectric with  $\varepsilon_r = 2.25$  while the region x < 0 is a free space. At the interface, subscript 1 denotes the field components on the +x side of the boundary and the subscript 2 on the -x side. If  $\overline{D}_1 = \hat{x} + 2\hat{y}$  C/m<sup>2</sup>, find  $\overline{D}_2$ ,  $\overline{E}_1$ , and  $\overline{E}_2$ 

### Problem 3

Verify the divergence theorem for a vector  $\overline{A} = \hat{r}5r$  when a closed surface is

- a. A cylinder with  $\rho = 1 \text{ m}$ , and  $0 \le z \le 1 \text{ m}$
- b. A Sphere of r = 1 m, and

A receiver antenna produces a voltage proportional to the y component of the incident electric field intensity. Assume that the proportionality constant is 0.25 m. If the electric field intensity is given by the following expression, then find the voltage induced at the receiver

$$\overline{E} = \hat{\phi} \frac{2 + \cos(\phi)}{\sqrt{\rho}} V / m$$

#### Problem 4

- a. A wireless communication network installed in the PG lecture room is allowed to use a  $10\ V/m$  radiation at 2.45 GHz.
  - 1. Find the power density in students, who are likely to use the room, if the wave is incident normally,
  - 2. Find the depth over which the field decreases by  $\frac{1}{2}$ .

Assume that the student's body can be modeled as a semi-infinite plane medium with  $\varepsilon_r = 47$  and  $\sigma = 2.21$  S/m and that the radiation is in the form of a uniform plane wave.

- 3. How do these results compare if the radiation frequency decreases to 40 MHz ( $\varepsilon_r = 97$  and  $\sigma = 0.7$  S/m) at this frequency?
- b. The Department decides to establish a wireless network in the PG lecture room using a 5.6 GHz signal. At the same time, the Department decides to re-furnish the furniture in the PG lecture room and these are to made from wooden boards from Iroko wood ( $\varepsilon_r = 2.1$ ).
  - 1. Find the appropriate thickness of the boards that keeps the furniture (assume partitions) from affecting the signal strength.

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Assume that the network uses uniform plane waves.

#### Problem 5

- a) An infinite filament on the *z* axis carries  $20\pi$  mA in the  $a_z$  direction. Three uniform cylindrical current sheets are also presents: 400 mA/m at  $\rho = 1$  cm, -250 mA/m at  $\rho = 2$  cm, and -300 mA/m at  $\rho = 3$  cm. Calculate  $H_{\phi}$  at  $\rho = 0.5$ , 1.5, 2.5, and 3.5 cm.
- a) Consider these regions in which  $\varepsilon' = 0$ : region 1, z < 0,  $\mu_1 = 4 \mu H/m$ and  $\varepsilon' = 10 \text{ pF/m}$ ; region 2, 0 < z < 6 cm,  $\mu_2 = 2 \mu H/m$ ,  $\varepsilon'_2 = 25 \text{ pF/m}$ ; region 3, z > 6 cm,  $\mu_3 = \mu_1$  and  $\varepsilon'_3 = \varepsilon'_1$ . What is the lowest frequency at which a uniform plane wave incident from region 1 onto the boundary at z = 0 will have no reflection?
  - If f = 50 MHz, what will the standing wave ratio be in region 1