Student Matriculation No:

Name:

# EEG 814: Electromagnetic Theory Assignment 

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## EEG 814 Electromagnetic Theory

## Problem 1

a. For a particular electromagnetic field, the Cartesian component of the electric field vector are given as:

$$
E_{x}=E_{y}=0 \quad E_{z}=E_{0} \cos (\alpha x) \cos (\omega t)
$$

Given that the magnetic field strength at time $t=0$ is $\bar{H}=0$, show that

$$
H_{x}=H_{z}=0 \quad H_{y}=H_{0} \sin (\alpha x) \sin (\omega t)
$$

1. Determine $H_{0}$ in terms of $E_{0}$ and the permeability $\mu$ of the medium in which the field exists.
b. The Cartesian $(x, y, z)$ components of the electric vector of a particular wave field propagating in an ideal dielectric medium of permeability $\mu_{0}$ and permittivity $\varepsilon$ are

$$
E_{x}=0 ; \quad E_{y}=E_{0} \sin (\omega t-\alpha x) ; \quad E_{z}=E_{0} \cos (\omega t-\alpha x)
$$

Where $E_{0}, \omega, \alpha$ are constants.

1. What is the state of polarization of this wave field?
2. Obtain expressions for the Cartesian component of the magnetic field strength, $\bar{H}$.
3. Show that the Poynting vector for the wave is independent of time and the spatial co-ordinates

## Problem 2

a. Two submerged submarines are using a 10 kHz plane electromagnetic wave for their communication. The magnitude of the electric field at the transmitter is $100 \mathrm{mV} / \mathrm{m}$, whereas the receiver requires at least $1 \mathrm{mV} / \mathrm{m}$ (peak value) for reliable communication. Assuming that the conductivity and the dielectric constant of the seawater are $4 \mathrm{~S} / \mathrm{m}$ and 81, respectively, find

1. The wavelength
2. The attenuation constant
3. The phase velocity
4. The skin depth of the wave, and
5. The maximum range over which a reliable communication is possible

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b. The region $x>0$ is a perfect dielectric with $\varepsilon_{r}=2.25$ while the region $x<0$ is a free space. At the interface, subscript 1 denotes the field components on the $+x$ side of the boundary and the subscript 2 on the $-x$ side. If $\bar{D}_{1}=\hat{x}+2 \hat{y} \mathrm{C} / \mathrm{m}^{2}$, find $\bar{D}_{2}, \bar{E}_{1}$, and $\bar{E}_{2}$

## Problem 3

Verify the divergence theorem for a vector $\bar{A}=\hat{r} 5 r$ when a closed surface is
a. A cylinder with $\rho=1 \mathrm{~m}$, and $0 \leq z \leq 1 \mathrm{~m}$
b. A Sphere of $r=1 \mathrm{~m}$, and

A receiver antenna produces a voltage proportional to the $y$ component of the incident electric field intensity. Assume that the proportionality constant is 0.25 m . If the electric field intensity is given by the following expression, then find the voltage induced at the receiver

$$
\bar{E}=\hat{\phi} \frac{2+\cos (\phi)}{\sqrt{\rho}} V / m
$$

## Problem 4

a. A wireless communication network installed in the PG lecture room is allowed to use a $10 \mathrm{~V} / \mathrm{m}$ radiation at 2.45 GHz .

1. Find the power density in students, who are likely to use the room, if the wave is incident normally,
2. Find the depth over which the field decreases by $1 / e$.

Assume that the student's body can be modeled as a semi-infinite plane medium with $\varepsilon_{r}=47$ and $\sigma=2.21 \mathrm{~S} / \mathrm{m}$ and that the radiation is in the form of a uniform plane wave.
3. How do these results compare if the radiation frequency decreases to $40 \mathrm{MHz} \quad\left(\varepsilon_{r}=97\right.$ and $\left.\sigma=0.7 \mathrm{~S} / \mathrm{m}\right) \quad$ at this frequency?
b. The Department decides to establish a wireless network in the PG lecture room using a 5.6 GHz signal. At the same time, the Department decides to re-furnish the furniture in the PG lecture room and these are to made from wooden boards from Iroko wood ( $\varepsilon_{r}=2.1$ ).

1. Find the appropriate thickness of the boards that keeps the furniture (assume partitions) from affecting the signal strength.

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Assume that the network uses uniform plane waves.

Problem 5
a) An infinite filament on the $z$ axis carries $20 \pi \mathrm{~mA}$ in the $a_{z}$ direction. Three uniform cylindrical current sheets are also presents: $400 \mathrm{~mA} / \mathrm{m}$ at $\rho=1 \mathrm{~cm},-250 \mathrm{~mA} / \mathrm{m}$ at $\rho=2 \mathrm{~cm}$, and $-300 \mathrm{~mA} / \mathrm{m}$ at $\rho=3 \mathrm{~cm}$. Calculate $H_{\phi}$ at $\rho=0.5,1.5,2.5$, and 3.5 cm .
a) Consider these regions in which $\varepsilon^{\prime \prime}=0$ : region $1, z<0, \mu_{1}=4 \mu \mathrm{H} / \mathrm{m}$ and $\varepsilon^{\prime}=10 \mathrm{pF} / \mathrm{m}$; region $2,0<z<6 \mathrm{~cm}, \mu_{2}=2 \mu \mathrm{H} / \mathrm{m}, \varepsilon_{2}^{\prime}=25 \mathrm{pF} / \mathrm{m}$; region $3, z>6 \mathrm{~cm}, \mu_{3}=\mu_{1}$ and $\varepsilon_{3}^{\prime}=\varepsilon_{1}^{\prime}$.
What is the lowest frequency at which a uniform plane wave incident from region 1 onto the boundary at $z=0$ will have no reflection?

If $f=50 \mathrm{MHz}$, what will the standing wave ratio be in region 1

