Vector Calculus Revision

1. Show that $\nabla \bullet \nabla \times \underline{\mathbf{A}} = 0$ (div • curl $\underline{\mathbf{A}} = 0$) for any vector of the form $\underline{\mathbf{A}} = \underline{\mathbf{i}}A_x + \underline{\mathbf{j}}A_y + \underline{\mathbf{k}}A_z$.

2. Show that $\nabla \times \nabla\varphi = 0$ (curl grad $\varphi = 0$) for any scalar field $\varphi(x,y,z)$.

3. Consider the position vector $\underline{\mathbf{r}}$ from the point (x_1,y_1,z_1) to the point (x,y,z) and show that $\nabla(1/r) = \text{grad}(1/r) = -\underline{\mathbf{r}}/r^3$

Gauss' Law and **E** Fields

4. In the equilibrium configuration, a spherical perfectly conducting shell of inner shell of inner radius *a* and outer radius *b* has a charge *q* fixed at the centre and a charge density σ uniformly distributed on the outer surface. Find the electric field for all *r* and the charge on the inner surface. Sketch the electric field pattern.

Electrostatics in dielectric media

5. A point charge Q is placed at the centre of a cavity of radius *a* within a concentric dielectric sphere of radius *b* and relative permittivity ε_r. Obtain formulae for the electric displacement $\underline{\mathbf{D}}$, electric field strength $\underline{\mathbf{E}}$ and the electrical potential V at a distant *r* from Q for $r < a$, $a < r < b$ and $r > b$.

6. A point charge Q is placed at the centre of a cavity of radius a within a concentric dielectric sphere of radius b and relative permittivity ε_r. Show that the total polarisation charge on the surface of radius *a* is $Q(1-\varepsilon_r)/\varepsilon_r$.

Dielectrics and Capacitors

7. A constant potential difference of 100 V is applied across the plates of a parallel-plate capacitor filled with a dielectric of relative permittivity $\varepsilon_r = 2$. If the plate separation is 0.1 mm, calculate the surface polarisation charge density on the two surfaces of the dielectric assuming that an infinitely small gap exists between these surfaces and the metal plates of the capacitor. The effects of fringing fields should be neglected. Why is the volume density of polarisation charge zero?

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8. A cylindrical capacitor is constructed with copper conductors; an inner conductor of radius *a* and a coaxial outer conductor of radius *b*. The space between the conductors is filled with an ideal dielectric material of relative permittivity ε_r. If λ is the charge per unit length on the inner conductor, derive relations for the fields **D** and **E**, and the polarisation **P** in the dielectric and for the polarisation charge density on the inner and outer surfaces of the dielectric. If $a=0.5$ mm, $b=3$ mm, ε_r =2.25 and the breakdown field strength (or dielectric strength) of the dielectric is 2×10^7 Vm^{-1}, calculate the maximum value that λ may have if dielectric breakdown is not to occur.

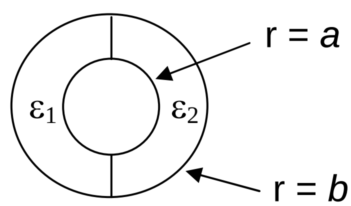
9. A capacitor is constructed with two metal coaxial cylinders, an inner cylinder of radius *a* and an outer cylinder of radius *b*. The space between the cylinders is filled with an inhomogeneous dielectric. If the relative permittivity ε_r varies with radial distance *R* (measured from the axis of symmetry) according to the equation $\varepsilon_r = b/R$, show that the capacitance per unit length of the capacitor is $2\pi\varepsilon_0\, b/(b-a)$.

10. A parallel-plate capacitor is charged to a potential *V* and then disconnected from the charging circuit. How much work is done by slowly changing the separation of the plates from *d* to *d'* ? (The plates are circular with radius $r \gg d$.)

Difficult electrostatics in dielectric media (more difficult than in the exam)!

11. One half of the region between the plates of a spherical capacitor of inner radius *a* and outer radius *b* is filled with a linear isotropic dielectric of permittivity ε_1 and the other half is filled with a linear isotropic dielectric of permittivity ε_2. See the sketch below. If the inner plate has total charge +Q and the outer plate total charge –Q then find:

(a) the electric displacements $\mathbf{D}_1$ and $\mathbf{D}_2$ in the region of ε_1 and ε_2;
(b) the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ in the region of ε_1 and ε_2;
(c) the total capacitance of the system.



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