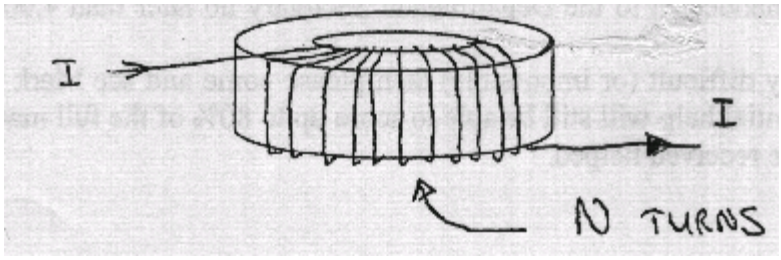
Electromagnetic Theory Question Sheet 2: $\underline{\mathbf{B}}$, $\underline{\mathbf{H}}$ and $\underline{\mathbf{M}}$

1. Consider 3 straight, infinitely long, equally spaced wires (spacing of the wires is d), each carrying a current I in the positive *x*-direction (the thickness of the wires can be assumed to be very much less than *d*).
   (a) Calculate the location of the two zeros in the magnetic field.
   (b) Sketch the magnetic field line pattern.
   (c) If the middle wire is rigidly displaced a very small distance z ($z << d$) upward while the other two wires are held fixed, describe the subsequent motion of the middle wire when released.

2. A long non-magnetic cylindrical conductor ($\mu_r = 1$) with inner radius a and outer radius b carries a current I. The current density in the conductor is uniform. Find the magnetic field set up by this current as a function of radius, inside the hollow space $(r < a)$, within the conductor $(a < r < b)$ and outside the conductor $(r > b)$.

3. A toroid having a soft iron core of square cross section and relative permeability μ_r is wound with N closely spaced turns of wire carrying a current I. Using Ampere's Law derive an expression for the magnetisation $\underline{\mathbf{M}}$ inside the iron.



4. A long straight wire is situated in free space. The axis of the wire defines the *z*-direction of the cylindrical polar coordinate system (r, ϕ, z). The wire is of radius a and relative permeability $\mu_r = 1001$ and it carries a current $I = 10\,\text{A}$ that is uniformly distributed over the cross section of the wire.

   (a) What is the direction of the magnetic field strength inside and outside the wire?

   (b) Find equations for the cylindrical polar components of the field strength $\underline{\mathbf{H}}$ and the magnetic flux density $\underline{\mathbf{B}}$ for $0 < r < \infty$. Also show that the magnetisation $\underline{\mathbf{M}}$ of the substance of the wire is given by the expression

$$\underline{\mathbf{M}} = \hat{\phi}\left(\frac{\mu_r - 1}{2\pi a^2}\right) r I$$

   for $r < a$.

   (c) Sketch graphs showing $\underline{\mathbf{H}}$, $\underline{\mathbf{B}}$ and $\underline{\mathbf{M}}$ as a function of r for the parameters given in the question.

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