Electromagnetic Theory EEG 814

Problem sheet 1

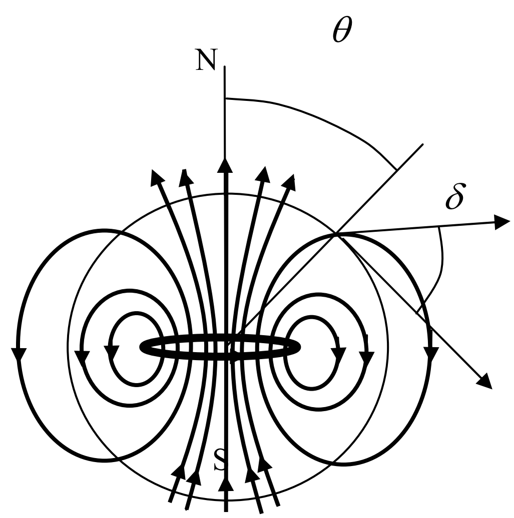
1. A plane wave in three dimensions can be represented by the expression

$$X(\mathbf{r},t) = X_0 \exp j(\mathbf{k}.\mathbf{r} - \omega t + \phi)$$

where $\mathbf{r}$ is a position vector to the general point (x,y,z)
$\mathbf{k}$ is the wave propagation vector
ω is the angular frequency

a) Show that the wavefronts (planes of constant X) are perpendicular to **k.**
b) If $t \to t + \delta t$ show, with the help of a sketch of $\text{Re}(X(\mathbf{r},t))$ as a function of $\hat{\mathbf{k}}.\mathbf{r}$, that the wave moves in the direction of +**k** with phase velocity $|\mathbf{v}_p| = \frac{\omega}{k}$.

2. Assuming that the earth's magnetic field is the same as that of a small magnetic dipole situated at the centre of the earth with its axis through the geographical poles, show that the angle of dip δ of the field lines with respect to the horizontal at a point on the surface of the earth an angle θ from the North pole is given by



$$\tan\delta = 2\cot\theta.$$ [5]

What is the dip angle for a point at latitude $\lambda = 30^0$? [5]

[Remember a) that latitude $\pm\lambda$ is measured from the equator, not the pole.
b) that the shape of the magnetic dipole field at long distances is identical to that of the electric dipole field.]

3. Use appropriate defining equations to derive the dimensions of the following electromagnetic quantities in terms of the basic dimensions of mass [*M*], length [*L*], time [*T*] and charge [*Q*]. (e.g. definition of current *I* can come from the equation $I = \frac{dQ}{dt}$, so its dimensions $[I] = [Q][T]^{-1}$)

electric field strength **E**	electric displacement **D**	[4]
….electrical polarization **P**	electrical susceptibility χ_e	[4]
permittivity of free space ε_0		[2]

4. A parallel plate capacitor has a gap thickness of 0.05 mm and an area of 0.5 square metre. A 1 volt potential is maintained across it. Calculate the charge *Q* on its plates and the size of the **E**, **D** and **P** fields in the gap if it is initially filled with vacuum [2.5]

How do these four values change when, at fixed voltage, either:

a) the gap is opened up to 0.1 mm, still filled with vacuum? [2.5]

b) the gap is kept at 0.05mm and filled with plastic, relative permittivity $\varepsilon_r = 5$? [2.5]

In the latter case, what is the free charge density on the plates, and what is the polarization charge density on the surface of the dielectric? [2.5]